

# thm\_2Epred\_\_set\_2EDELETE\_\_SUBSET (TMUzJ8DthoFXJT7MY4ZJY6BbWhRfHq51mu8)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(\lambda V1x \in 2.V0x)))$

**Definition 4** We define  $c\_2Ebool\_2EET$  to be  $(\lambda p (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(\lambda V1x \in 2.V0x.(c\_2Emin\_2E\_3D (2^{A\_27a}) (V1x P))))$

**Definition 6** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.(\lambda V0s \in (2^{A\_27a}).(\lambda V1t \in (2^{A\_27a}).(\lambda V1x \in 2.V0x.(c\_2Emin\_2E\_3D (2^{A\_27a}) (V1x s)))))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F$  to be  $(\lambda p (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 8** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap (c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F)))$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V0t2.(c\_2Emin\_2E\_3D (2^{A\_27a}) (V2t t1))))))$

**Definition 10** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E\_2F)$ .

**Definition 11** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V0t2.(c\_2Emin\_2E\_3D (2^{A\_27a}) (V2t t1))))))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 12** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Emin\_2E\_3D (2^{A\_27a}) (V1x V0x)))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (3)$$

**Definition 13** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A\_27a}). (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1s)$

**Definition 14** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1t)$

**Definition 15** We define  $c\_2Epred\_set\_2EDELETE$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1x \in A\_27a. (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1x)$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1x \in \\ A\_27a. (\forall V2y \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1x) \\ (ap\ (ap\ (c\_2Epred\_set\_2EDELETE\ A\_27a)\ V0s)\ V2y))) \Leftrightarrow ((p\ (ap\ (ap \\ (c\_2Ebool\_2EIN\ A\_27a)\ V1x)\ V0s)) \wedge (\neg(V1x = V2y))))))) \end{aligned} \quad (4)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1s \in \\ (2^{A\_27a}). (p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ A\_27a)\ (ap\ (ap\ (c\_2Epred\_set\_2EDELETE \\ A\_27a)\ V1s)\ V0x))\ V1s)))) \end{aligned}$$