

thm_2Epred__set_2EDISJOINT__ALT (TMTGrGHzRXfHikcQtSXXjTrtanwMijayUc2)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2IN$ to be $\lambda A.\lambda a : \iota.(\lambda V0x \in A.\lambda a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

Definition 4 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 5 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27$

Definition 7 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A.\lambda a : \iota.(\lambda V0x \in A.\lambda a.c_2Ebool_2E_2F)$.

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda a : \iota.(\lambda A.\lambda b : \iota.(\lambda V0x \in A.\lambda a.\lambda V1y \in A.\lambda b.(ap (c_2Emin_2E_3D (2^{A-27a}).(ap V1y V0x)) (c_2Emin_2E_3D (2^{A-27b}).(ap V0x V1y)))) \tag{2}$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda V0x \in A.\lambda a.\lambda V1y \in A.\lambda b.(ap (c_2Emin_2E_3D (2^{A-27a}).(ap V1y V0x)) (c_2Emin_2E_3D (2^{A-27b}).(ap V0x V1y))))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \end{aligned} \quad (3)$$

Definition 12 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Epred_set_2EINTER\ V0s)\ V1t)$

Definition 13 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Epred_set_2EDISJOINT\ V0s)\ V1t)$

Definition 14 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (V0t1\ V1t2))))$

Definition 15 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E\ V0t))$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (5)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (7)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ p\ V0t)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\exists V1x \in A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\exists V2x \in A_27a.((p\ (ap\ V0P\ V2x)) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\exists V3x \in A_27a.(p\ (ap\ V0P\ V3x))) \vee (\exists V4x \in A_27a.(p\ (ap\ V1Q\ V4x))))))) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.((\exists V2x \in A_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A_27a.(p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).((\exists V2x \in A_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \wedge (\exists V3x \in A_27a.(p\ (ap\ V1Q\ V3x))))))) \quad (16)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in (2^{A_27a}).((p\ (ap\ (ap\ (c_2Epred_set_2EDISJOINT\ A_27a)\ V0s)\ V1t)) \Leftrightarrow (\neg(\exists V2x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t))))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (19)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (20)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (21)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (22)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p \\ & V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (24) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (25) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (26) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (27) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (28) \end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in \\ & (2^{A_27a}).((p\ (ap\ (ap\ (c_2Epred_set_2EDISJOINT\ A_27a)\ V0s)\ V1t)) \Leftrightarrow \\ & (\forall V2x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \Rightarrow \\ & (\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))) \end{aligned}$$