

# thm\_2Epred\_\_set\_2EEXPLICIT\_\_ENUMERATE\_\_MONO (TMbnz2Cu4vYsWaf6acXVgNby4hyb3hzHTAb)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $c\_2Earithmetic\_2EFUNPOW : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Earithmetic\_2EFUNPOW\ A\_27a \in \\ (((A\_27a^{A\_27a})^{ty\_2Enum\_2Enum})^{(A\_27a^{A\_27a})}) \quad (2)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2. V0x))\ (\lambda V1x \in 2. V1x))$

**Definition 3** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V0x))$

**Definition 4** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. (\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 5** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota. (ap\ (ap\ (c\_2Ecombin\_2ES\ A\_27a\ (A\_27a^{A\_27a}))\ A\_27a))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (3)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (4)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (5)$$

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a})))$

**Definition 7** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$   
Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{6}$$

**Definition 8** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 9** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Let  $c\_2Epred\_set\_2ECHOICE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epred\_set\_2ECHOICE\ A\_27a \in (A\_27a^{(2^{A\_27a})}) \tag{7}$$

**Definition 10** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 11** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 12** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$   
of type  $\iota$ .

**Definition 13** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$

**Definition 14** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{8}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{9}$$

**Definition 15** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Emin\_2E\_3D\_3D\_3E\ V0x\ V1y))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{10}$$

**Definition 16** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_2Emin\_2E\_3D\_3D\_3E\ V0x\ V1s))$

**Definition 17** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF))$

**Definition 18** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2Emin\_2E\_3D\_3D\_3E\ V0s\ V1t))$

**Definition 19** We define  $c\_2Epred\_set\_2EDELETE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1x \in A\_27a.(ap (ap$

**Definition 20** We define  $c\_2Epred\_set\_2EREST$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (ap (c\_2Epred\_set\_2E$

**Definition 21** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap ($

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow ((\forall V0f \in (A\_27a^{A-27a}).(\forall V1x \in \\ & A\_27a.((ap (ap (ap (c\_2Earithmetic\_2EFUNPOW A\_27a) V0f) c\_2Enum\_2E0) \\ & V1x) = V1x)) \wedge (\forall V2f \in (A\_27a^{A-27a}).(\forall V3n \in ty\_2Enum\_2Enum. \\ & (\forall V4x \in A\_27a.((ap (ap (ap (c\_2Earithmetic\_2EFUNPOW A\_27a) \\ & V2f) (ap c\_2Enum\_2ESUC V3n)) V4x) = (ap (ap (ap (c\_2Earithmetic\_2EFUNPOW \\ & A\_27a) V2f) V3n) (ap V2f V4x))))))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0f \in (A\_27a^{A-27a}).(\forall V1n \in \\ & ty\_2Enum\_2Enum.(\forall V2x \in A\_27a.((ap (ap (ap (c\_2Earithmetic\_2EFUNPOW \\ & A\_27a) V0f) (ap c\_2Enum\_2ESUC V1n)) V2x) = (ap V0f (ap (ap (c\_2Earithmetic\_2EFUNPOW \\ & A\_27a) V0f) V1n) V2x)))))) \end{aligned} \quad (12)$$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (14)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg (p V0t)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (19)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (22)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C))))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (25)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (26)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))))) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap (c.2Ecombin\_2EI A\_27a) V0x) = V0x)) \quad (28)$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap V0P c\_2Enum\_2E0))\wedge \\
& (\forall V1n \in ty\_2Enum\_2Enum.((p (ap V0P V1n))\Rightarrow(p (ap V0P (ap c\_2Enum\_2ESUC \\
& V1n))))))\Rightarrow(\forall V2n \in ty\_2Enum\_2Enum.(p (ap V0P V2n))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a\Rightarrow(\forall V0s \in (2^{A\_27a}).(\forall V1t \in \\
& (2^{A\_27a}).(\forall V2u \in (2^{A\_27a}).(((p (ap (ap (c\_2Epred\_set\_2ESUBSET \\
& A\_27a) V0s) V1t))\wedge(p (ap (ap (c\_2Epred\_set\_2ESUBSET A\_27a) V1t) \\
& V2u))))\Rightarrow(p (ap (ap (c\_2Epred\_set\_2ESUBSET A\_27a) V0s) V2u))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a\Rightarrow(\forall V0s \in (2^{A\_27a}).(p (ap ( \\
& ap (c\_2Epred\_set\_2ESUBSET A\_27a) (ap (c\_2Epred\_set\_2EREST \\
& A\_27a) V0s)) V0s)))
\end{aligned} \tag{31}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t)))\Leftrightarrow(p V0t))) \tag{32}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A)\Rightarrow((\neg(p V0A))\Rightarrow False))) \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)\vee(p V1B)))\Rightarrow False)\Leftrightarrow \\
& (((p V0A)\Rightarrow False)\Rightarrow((\neg(p V1B))\Rightarrow False))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(\neg(p V0A))\vee(p V1B)))\Rightarrow False)\Leftrightarrow \\
& ((p V0A)\Rightarrow((\neg(p V1B))\Rightarrow False))))))
\end{aligned} \tag{35}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A))\Rightarrow False)\Rightarrow(((p V0A)\Rightarrow False)\Rightarrow False))) \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p)\Leftrightarrow( \\
& (p V1q)\Leftrightarrow(p V2r)))\Leftrightarrow(((p V0p)\vee((p V1q)\vee(p V2r)))\wedge(((p V0p)\vee((\neg( \\
& p V2r))\vee(\neg(p V1q))))\wedge(((p V1q)\vee((\neg(p V2r))\vee(\neg(p V0p))))\wedge((p V2r)\vee \\
& ((\neg(p V1q))\vee(\neg(p V0p))))))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ( \\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{41}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0n \in \text{ty\_2Enum\_2Enum.} ( \\
& \forall V1s \in (2^{A\_27a}). (p \text{ (ap (ap (c\_2Epred\_set\_2ESUBSET } A\_27a) \\
& (ap \text{ (ap (ap (c\_2Earithmetic\_2EFUNPOW (2}^{A\_27a}) \text{ (c\_2Epred\_set\_2EREST} \\
& A\_27a)) V0n) V1s)) V1s))))
\end{aligned}$$