

# thm\_2Epred\_\_set\_2EEXPLICIT\_\_ENUMERATE\_\_NOT\_\_EMPTY (TMWWgm8A7DLUhJpJNt652X2Kdahr5pgMZd8)

October 26, 2020

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $c\_2Earithmetic\_2EFUNPOW : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Earithmetic\_2EFUNPOW\ A\_27a \in \\ (((A\_27a^{A\_27a})^{ty\_2Enum\_2Enum})^{(A\_27a^{A\_27a})}) \quad (2)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V0x))$

**Definition 4** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. (\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 5** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota. (ap\ (ap\ (c\_2Ecombin\_2ES\ A\_27a\ (A\_27a^{A\_27a}))\ A\_27a))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (3)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (4)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (5)$$

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a})))$

**Definition 7** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ ($   
Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{6}$$

**Definition 8** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 9** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 10** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

Let  $c\_2Epred\_set\_2ECHOICE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epred\_set\_2ECHOICE\ A\_27a \in (A\_27a^{(2^{A\_27a})}) \tag{7}$$

**Definition 11** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

**Definition 12** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$   
of type  $\iota$ .

**Definition 13** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 14** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{8}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{9}$$

**Definition 15** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{10}$$

**Definition 16** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_2$

**Definition 17** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E$

**Definition 18** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

**Definition 19** We define `c_2Epred_set_2EDELETE` to be  $\lambda A_{.27a} : \iota.\lambda V0s \in (2^{A_{.27a}}).\lambda V1x \in A_{.27a}.(ap (ap$

**Definition 20** We define `c_2Epred_set_2EREST` to be  $\lambda A_{.27a} : \iota.\lambda V0s \in (2^{A_{.27a}}).(ap (ap (c_2Epred\_set\_2E0$

**Definition 21** We define `c_2Epred_set_2EFINITE` to be  $\lambda A_{.27a} : \iota.\lambda V0s \in (2^{A_{.27a}}).(ap (c_2Ebool\_2E\_21 (2$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow ((\forall V0f \in (A_{.27a}^{A_{.27a}}).(\forall V1x \in \\ & A_{.27a}.(ap (ap (ap (c_2Earithmetic_2EFUNPOW A_{.27a}) V0f) c_2Enum_2E0 \\ & V1x) = V1x))) \wedge (\forall V2f \in (A_{.27a}^{A_{.27a}}).(\forall V3n \in ty_2Enum_2Enum. \\ & (\forall V4x \in A_{.27a}.(ap (ap (ap (c_2Earithmetic_2EFUNPOW A_{.27a}) \\ & V2f) (ap c_2Enum_2ESUC V3n)) V4x) = (ap (ap (ap (c_2Earithmetic_2EFUNPOW \\ & A_{.27a}) V2f) V3n) (ap V2f V4x))))))))) \end{aligned} \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \vee \neg(p V0t)))) \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ & A_{.27a}.(p V0t)) \Leftrightarrow (p V0t))) \end{aligned} \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\
& True))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\
& A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee ( \\
& (p \ V1B) \vee (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee \\
& (p \ V0A))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg( \\
& p \ V0A)) \vee (\neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_{27} \in 2. (\forall V2y \in 2. (\forall V3y_{27} \in \\ & 2. (((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \end{aligned} \quad (30)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0x \in A_{27a}. ((\text{ap } (c\_2Ecombin\_2EI \\ A_{27a}) V0x) = V0x)) \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty\_2Enum\_2Enum}). (((p (\text{ap } V0P \ c\_2Enum\_2E0)) \wedge \\ & (\forall V1n \in ty\_2Enum\_2Enum. ((p (\text{ap } V0P \ V1n)) \Rightarrow (p (\text{ap } V0P \ (\text{ap } c\_2Enum\_2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum. (p (\text{ap } V0P \ V2n)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (p (\text{ap } (c\_2Epred\_set\_2EFINITE \\ A_{27a}) (c\_2Epred\_set\_2EEMPTY \ A_{27a}))) \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0s \in (2^{A_{27a}}). ((p (\text{ap} \\ & (c\_2Epred\_set\_2EFINITE \ A_{27a}) (\text{ap } (c\_2Epred\_set\_2EREST \ A_{27a}) \\ & V0s))) \Leftrightarrow (p (\text{ap } (c\_2Epred\_set\_2EFINITE \ A_{27a}) V0s)))) \end{aligned} \quad (34)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\ & (((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\ & ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \end{aligned} \quad (38)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (39)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg( \\
& p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee \\
& ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p)))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (( \\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{44}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{45}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{46}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \tag{47}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{48}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{49}$$

**Theorem 1**

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0n \in ty.2Enum.2Enum.( \\
& \forall V1s \in (2^{A.27a}). ((\neg(p \ (ap \ (c.2Epred\_set.2EFINITE \ A.27a) \\
& V1s))) \Rightarrow (\neg((ap \ (ap \ (ap \ (c.2Earithmetic.2EFUNPOW \ (2^{A.27a})) \ (c.2Epred\_set.2EREST \\
& A.27a)) \ V0n) \ V1s) = (c.2Epred\_set.2EEMPTY \ A.27a))))))
\end{aligned}$$