

# thm\_2Epred\_set\_2EFINITELY\_INJECTIVE\_IMAGE\_FINITE (TMFJb2g8ScTeZ3Wd3b2LA4o8rCjxc99VuqH)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$   
of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 3** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

**Definition 6** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 7** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a}))$

**Definition 8** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$   
of type  $\iota$ .

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (2)$$

**Definition 10** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \end{aligned} \quad (3)$$

**Definition 11** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ **if**  $(\exists x \in A.p\ (ap\ P\ x))$  **then** *(the  $(\lambda x.x \in A \wedge p\ x)$  of type  $\iota \Rightarrow \iota$ ).*

**Definition 12** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ V0P))))$

**Definition 13** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

**Definition 14** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ V2t))))$

**Definition 15** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E\_3F\ V1s\ x))$

**Definition 16** We define  $c\_2Ebool\_2E\_F$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 17** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E\_F)$ .

**Definition 18** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_3F))$

**Definition 19** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E\_3F\ V1t\ s))$

**Definition 20** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A\_27b}).(ap\ (c\_2Ebool\_2E\_3F\ V1s\ f))$

**Definition 21** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E\_21\ 2)\ V0s))$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (5)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee \neg(p\ V0t))) \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (8)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \wedge (p \ V1t2) \wedge (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \wedge (p \ V2t3)))))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow \text{False}) \Rightarrow (\neg(p \ V0t)))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow \text{False}))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge \text{True}) \Leftrightarrow (p \ V0t)) \wedge (((\text{False} \wedge (p \ V0t)) \Leftrightarrow \text{False}) \wedge (((p \ V0t) \wedge \text{False}) \Leftrightarrow \text{False}) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \vee (p \ V0t)) \Leftrightarrow \text{True}) \wedge (((p \ V0t) \vee \text{True}) \Leftrightarrow \text{True}) \wedge (((\text{False} \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \text{False}) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow \text{True}) \Leftrightarrow \text{True}) \wedge (((\text{False} \Rightarrow (p \ V0t)) \Leftrightarrow \text{True}) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow \text{True}) \wedge (((p \ V0t) \Rightarrow \text{False}) \Leftrightarrow (\neg(p \ V0t)))))) \quad (14)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg \text{True}) \Leftrightarrow \text{False}) \wedge ((\neg \text{False}) \Leftrightarrow \text{True}))) \quad (15)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \text{True})) \quad (16)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow \text{True}) \Leftrightarrow (p \ V0t)) \wedge (((\text{False} \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow \text{False}) \Leftrightarrow (\neg(p \ V0t)))))) \quad (18)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).((\neg(\exists V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (19)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A-27a}).((p V0P) \wedge (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p V0P) \wedge (p (ap V1Q V3x)))))) \quad (20)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1Q \in 2.(((\exists V2x \in A.27a.(p (ap V0P V2x))) \vee (p V1Q)) \Leftrightarrow (\exists V3x \in A.27a.((p (ap V0P V3x)) \vee (p V1Q)))))) \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A-27a}).(((p V0P) \vee (\exists V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A.27a.((p V0P) \vee (p (ap V1Q V3x)))))) \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1Q \in 2.((\exists V2x \in A.27a.((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A.27a.(p (ap V0P V3x))) \wedge (p V1Q)))))) \quad (23)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A-27a}).(((\exists V2x \in A.27a.((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A.27a.(p (ap V1Q V3x)))))) \quad (24)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A-27a}).(((\forall V2x \in A.27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A.27a.(p (ap V1P V3x)) \vee (p V0Q)))))) \quad (25)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A-27a}).(((\forall V2x \in A.27a.((p V0P) \Rightarrow (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \Rightarrow (\forall V3x \in A.27a.(p (ap V1Q V3x)))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (31)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (32)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1)))))) \quad (33)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (34)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (35)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0f \in (2^{A_{.27a}}).(\forall V1v \in A_{.27a}.((\forall V2x \in A_{.27a}.((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (ap V0f V1v)))))) \quad (36)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0P \in ((2^{A_{.27b}})^{A_{.27a}}).((\forall V1x \in A_{.27a}.(\exists V2y \in A_{.27b}.(p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A_{.27b}^{A_{.27a}}).(\forall V4x \in A_{.27a}.(p (ap (ap V0P V4x) (ap V3f V4x)))))) \quad (37)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap\ (c\_2Ecombin\_2EI\ A\_27a)\ V0x) = V0x)) \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0x \in A\_27a. (\forall V1y \in A\_27b. (\forall V2a \in A\_27a. (\forall V3b \in \\ & \quad A\_27b. (((ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap \\ & \quad (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\ & \quad (2^{A\_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & \quad A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}). (\forall V1v \in \\ & \quad A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_set\_2EGSPEC \\ & \quad A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b. ((ap\ (ap\ (c\_2Epair\_2E\_2C \\ & \quad A\_27a\ 2)\ V1v)\ c\_2Ebool\_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\ & \quad (2^{A\_27a}). (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ \\ & \quad V2x)\ (ap\ (ap\ (c\_2Epred\_set\_2EDIFF\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ ( \\ & \quad ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V0s)) \wedge (\neg (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & \quad A\_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\ & \quad A\_27a. (\forall V2s \in (2^{A\_27a}). ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ \\ & \quad V0x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ A\_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = \\ & \quad V1y) \vee (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ V2s)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0y \in A\_27b. (\forall V1s \in (2^{A\_27a}). (\forall V2f \in (A\_27b^{A\_27a}). \\ & \quad ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27b)\ V0y)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\ & \quad A\_27a\ A\_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A\_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ & \quad (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V1s)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ \forall V0s \in (2^{A.27a}).(\forall V1f \in (A.27b^{A.27a}).(((ap\ (ap\ ( \\ c.2Epred\_set.2EIMAGE\ A.27a\ A.27b)\ V1f)\ V0s) = (c.2Epred\_set.2EEMPTY \\ A.27b)) \Leftrightarrow (V0s = (c.2Epred\_set.2EEMPTY\ A.27a)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (p\ (ap\ (c.2Epred\_set.2EFINITE\ A.27a)\ (c.2Epred\_set.2EEMPTY\ A.27a))) \quad (46)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(2^{A.27a})}).(( \\ (p\ (ap\ V0P\ (c.2Epred\_set.2EEMPTY\ A.27a))) \wedge (\forall V1s \in (2^{A.27a}). \\ ((p\ (ap\ (c.2Epred\_set.2EFINITE\ A.27a)\ V1s)) \wedge (p\ (ap\ V0P\ V1s)))) \Rightarrow \\ (\forall V2e \in A.27a.((\neg(p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2e)\ V1s)))) \Rightarrow \\ (p\ (ap\ V0P\ (ap\ (ap\ (c.2Epred\_set.2EINSERT\ A.27a)\ V2e)\ V1s)))))) \Rightarrow \\ (\forall V3s \in (2^{A.27a}).((p\ (ap\ (c.2Epred\_set.2EFINITE\ A.27a)\ V3s)) \Rightarrow (p\ (ap\ V0P\ V3s)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ (2^{A.27a}).(((p\ (ap\ (c.2Epred\_set.2EFINITE\ A.27a)\ (ap\ (ap\ (c.2Epred\_set.2EDIFF \\ A.27a)\ V0P)\ V1Q)))) \wedge (p\ (ap\ (c.2Epred\_set.2EFINITE\ A.27a)\ V1Q)))) \Rightarrow \\ (p\ (ap\ (c.2Epred\_set.2EFINITE\ A.27a)\ V0P)))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ \forall V0s \in (2^{A.27a}).((p\ (ap\ (c.2Epred\_set.2EFINITE\ A.27a)\ V0s)) \Rightarrow (\forall V1f \in (A.27b^{A.27a}).(p\ (ap\ (c.2Epred\_set.2EFINITE \\ A.27b)\ (ap\ (ap\ (c.2Epred\_set.2EIMAGE\ A.27a\ A.27b)\ V1f)\ V0s)))))) \end{aligned} \quad (49)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (51)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (52)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (53)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r))) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (62)$$



Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (64)$$

**Theorem 1**

$$\begin{aligned} & \forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty \ A_{.27b} \Rightarrow ( \\ & \forall V0f \in (A_{.27b}^{A_{.27a}}).((\forall V1x \in A_{.27b}.(p \ (ap \ (c_{.2}Epred\_set_{.2}EFINITE \\ & A_{.27a}) \ (ap \ (c_{.2}Epred\_set_{.2}EGSPEC \ A_{.27a} \ A_{.27a}) \ (\lambda V2y \in A_{.27a}. \\ & (ap \ (ap \ (c_{.2}Epair_{.2E.2C} \ A_{.27a} \ 2) \ V2y) \ (ap \ (ap \ (c_{.2}Emin_{.2E.3D} \ A_{.27b}) \\ & V1x) \ (ap \ V0f \ V2y)))))) \Rightarrow (\forall V3s \in (2^{A_{.27a}}).((p \ (ap \ (c_{.2}Epred\_set_{.2}EFINITE \\ & A_{.27b}) \ (ap \ (ap \ (c_{.2}Epred\_set_{.2}EIMAGE \ A_{.27a} \ A_{.27b}) \ V0f) \ V3s))) \Leftrightarrow \\ & (p \ (ap \ (c_{.2}Epred\_set_{.2}EFINITE \ A_{.27a}) \ V3s)))))) \end{aligned}$$