

thm\_2Epred\_set\_2EFINITE\_COUNT  
(TMQcqy9PYUznPk6aTztsimtcXYG5Uc2dWbe)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 8** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 9** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 10** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Emin\_2E\_3D (2^{A\_27a})$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \tag{3}$$

**Definition 11** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A.27a : \iota.\lambda V0x \in A.27a.\lambda V1s \in (2^{A-27a}).(ap (c\_2E$

**Definition 12** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A.27a : \iota.(\lambda V0x \in A.27a.c\_2Ebool\_2EF)$ .

**Definition 13** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c\_2Ebool\_2E21 (2$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{4}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{5}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{6}$$

**Definition 14** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{7}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{8}$$

**Definition 15** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

**Definition 16** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge$   
of type  $\iota \Rightarrow \iota$ .

**Definition 17** We define  $c\_2Ebool\_2E3F$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P (ap (c\_2Emin\_2E40$

**Definition 18** We define  $c\_2Eprim\_rec\_2E3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 19** We define  $c\_2Epred\_set\_2Ecount$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (c\_2Epred\_set\_2EG$

Assume the following.

$$True \tag{9}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p\ V0t)))))) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap\ V0P\ c\_2Enum\_2E0)) \wedge \\ & (\forall V1n \in ty\_2Enum\_2Enum.((p (ap\ V0P\ V1n)) \Rightarrow (p (ap\ V0P (ap\ c\_2Enum\_2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p (ap\ V0P\ V2n)))))) \end{aligned} \tag{11}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (p\ (ap\ (c.2Epred\_set.2EFINITE\ A.27a)\ (c.2Epred\_set.2EEMPTY\ A.27a))) \quad (12)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1s \in (2^{A.27a}). ((p\ (ap\ (c.2Epred\_set.2EFINITE\ A.27a)\ (ap\ (ap\ (c.2Epred\_set.2EINSERT\ A.27a)\ V0x)\ V1s))) \Leftrightarrow (p\ (ap\ (c.2Epred\_set.2EFINITE\ A.27a)\ V1s)))))) \quad (13)$$

Assume the following.

$$((ap\ c.2Epred\_set.2Ecount\ c.2Enum.2E0) = (c.2Epred\_set.2EEMPTY\ ty.2Enum.2Enum)) \quad (14)$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum. ((ap\ c.2Epred\_set.2Ecount\ (ap\ c.2Enum.2ESUC\ V0n)) = (ap\ (ap\ (c.2Epred\_set.2EINSERT\ ty.2Enum.2Enum)\ V0n)\ (ap\ c.2Epred\_set.2Ecount\ V0n)))) \quad (15)$$

**Theorem 1**

$$(\forall V0n \in ty.2Enum.2Enum. (p\ (ap\ (c.2Epred\_set.2EFINITE\ ty.2Enum.2Enum)\ (ap\ c.2Epred\_set.2Ecount\ V0n))))$$