

# thm\_2Epred\_set\_2EFINITE\_DELETE (TMSUAnaR9E7Pg4HaRrUzoxi5G8U89JbJsBL)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 6** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 9** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap (c\_2Emin\_2E\_40 (2^{A\_27a}))$

**Definition 10** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 11** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 12** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}})$$
(3)

**Definition 13** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_7E$

**Definition 14** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap (c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E$

**Definition 15** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_21$

**Definition 16** We define  $c\_2Epred\_set\_2EDELETE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap (ap (c\_2Ebool\_2E\_21$

**Definition 17** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_21 (2^{A\_27a})$

Assume the following.

$$True$$
(4)

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))$$
(5)

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t)))$$
(6)

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg (p V0t))))$$
(7)

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t)))$$
(8)

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p V0t))))))$$
(9)

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) V0t1) V1t2) = V1t2))))))$$
(10)

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1s \in \\ (2^{A_{.27a}}).((\neg(p\ (ap\ (ap\ (c_{.2Ebool\_2EIN}\ A_{.27a})\ V0x)\ V1s))) \Leftrightarrow ((ap\ (11) \\ (ap\ (c_{.2Epred\_set\_2EDELETE}\ A_{.27a})\ V1s)\ V0x) = V1s)))) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.((ap\ (ap\ (c_{.2Epred\_set\_2EDELETE} \\ A_{.27a})\ (c_{.2Epred\_set\_2EEMPTY}\ A_{.27a}))\ V0x) = (c_{.2Epred\_set\_2EEMPTY} \\ A_{.27a}))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in \\ A_{.27a}.(\forall V2s \in (2^{A_{.27a}}).((ap\ (ap\ (c_{.2Epred\_set\_2EDELETE} \\ A_{.27a})\ (ap\ (ap\ (c_{.2Epred\_set\_2EINSERT}\ A_{.27a})\ V0x)\ V2s))\ V1y) = \\ (ap\ (ap\ (ap\ (c_{.2Ebool\_2ECOND}\ (2^{A_{.27a}}))\ (ap\ (ap\ (c_{.2Emin\_2E\_3D} \\ A_{.27a})\ V0x)\ V1y))\ (ap\ (ap\ (c_{.2Epred\_set\_2EDELETE}\ A_{.27a})\ V2s)\ V1y)) \\ (ap\ (ap\ (c_{.2Epred\_set\_2EINSERT}\ A_{.27a})\ V0x)\ (ap\ (ap\ (c_{.2Epred\_set\_2EDELETE} \\ A_{.27a})\ V2s)\ V1y)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1s \in \\ (2^{A_{.27a}}).((p\ (ap\ (ap\ (c_{.2Ebool\_2EIN}\ A_{.27a})\ V0x)\ V1s)) \Rightarrow ((ap\ (ap \\ (c_{.2Epred\_set\_2EINSERT}\ A_{.27a})\ V0x)\ (ap\ (ap\ (c_{.2Epred\_set\_2EDELETE} \\ A_{.27a})\ V1s)\ V0x)) = V1s)))) \end{aligned} \quad (14)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (p\ (ap\ (c_{.2Epred\_set\_2EFINITE} \\ A_{.27a})\ (c_{.2Epred\_set\_2EEMPTY}\ A_{.27a}))) \quad (15)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0P \in (2^{(2^{A_{.27a}})}).(( \\ (p\ (ap\ V0P\ (c_{.2Epred\_set\_2EEMPTY}\ A_{.27a}))) \wedge (\forall V1s \in (2^{A_{.27a}}). \\ (((p\ (ap\ (c_{.2Epred\_set\_2EFINITE}\ A_{.27a})\ V1s)) \wedge (p\ (ap\ V0P\ V1s))) \Rightarrow \\ (\forall V2e \in A_{.27a}.((\neg(p\ (ap\ (ap\ (c_{.2Ebool\_2EIN}\ A_{.27a})\ V2e)\ V1s))) \Rightarrow \\ (p\ (ap\ V0P\ (ap\ (ap\ (c_{.2Epred\_set\_2EINSERT}\ A_{.27a})\ V2e)\ V1s)))))) \Rightarrow \\ (\forall V3s \in (2^{A_{.27a}}).((p\ (ap\ (c_{.2Epred\_set\_2EFINITE}\ A_{.27a}) \\ V3s)) \Rightarrow (p\ (ap\ V0P\ V3s)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1s \in \\ (2^{A_{.27a}}).((p\ (ap\ (c_{.2Epred\_set\_2EFINITE}\ A_{.27a})\ (ap\ (ap\ (c_{.2Epred\_set\_2EINSERT} \\ A_{.27a})\ V0x)\ V1s))) \Leftrightarrow (p\ (ap\ (c_{.2Epred\_set\_2EFINITE}\ A_{.27a})\ V1s)))) \end{aligned} \quad (17)$$

**Theorem 1**

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V_0 x \in A_{27a}. (\forall V_1 s \in (2^{A_{27a}}). ((p (ap (c\_2Epred\_set\_2EFINITE A_{27a}) (ap (ap (c\_2Epred\_set\_2EDELETE A_{27a}) V_1 s) V_0 x))) \Leftrightarrow (p (ap (c\_2Epred\_set\_2EFINITE A_{27a}) V_1 s))))))$$