

thm_2Epred_set_2EFINITE_ISO_NUM
(TMUBde4Xi89ExaqcQntZzdPm1q1i7K4KgNX)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x)))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A-27b})^{A-27a}}) \tag{2}$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_2F_5C (2^{A-27a}) (2^{A-27b})))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod A_27a 2)^{A-27b}}) \tag{3}$$

Definition 9 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ P))))$

Definition 12 We define $c_2Epred_set_2ESURJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27b})$

Definition 13 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27b})$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (4)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \quad (5)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \quad (6)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (7)$$

Definition 14 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Definition 15 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Epred_set_2ECARD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Epred_set_2ECARD\ A_27a \in (ty_2Enum_2Enum^{(2^{A_27a})}) \quad (8)$$

Definition 16 We define $c_2Epred_set_2Ecount$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (c_2Epred_set_2EG\ n))$

Definition 17 We define $c_2Epred_set_2EBIJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27b})$

Definition 18 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ t1\ t2)))$

Definition 19 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_21\ s\ x))$

Definition 20 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2F)$.

Definition 21 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_21\ s))$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (15)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))))) \Rightarrow (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27)))))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.(((p \ V1Q) \Rightarrow (\forall V2x \in A_27a.(p \ (ap \ V0P \ V2x)))) \Leftrightarrow (\forall V3x \in A_27a.((p \ V1Q) \Rightarrow (p \ (ap \ V0P \ V3x)))))) \wedge (((\forall V4x \in A_27a.(p \ (ap \ V0P \ V4x)) \wedge (p \ V1Q)) \Leftrightarrow (\forall V5x \in A_27a.((p \ (ap \ V0P \ V5x)) \wedge (p \ V1Q)))) \wedge (((p \ V1Q) \wedge (\forall V6x \in A_27a.(p \ (ap \ V0P \ V6x)))) \Leftrightarrow (\forall V7x \in A_27a.((p \ V1Q) \wedge (p \ (ap \ V0P \ V7x)))))))))) \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ & \quad A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ & \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b))) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\ & \hspace{15em} (18) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & \quad (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & \quad A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))) \\ & \hspace{15em} (19) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\ & \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ & \quad A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\ & \quad A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \\ & \hspace{15em} (20) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1y \in A_27a. (\forall V2x \in A_27b. \\ & \quad (\forall V3P \in (2^{A_27a}). (((p\ (ap\ V3P\ V1y)) \wedge (V2x = (ap\ V0f\ V1y))) \Rightarrow \\ & \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V2x)\ (ap\ (c_2Epred_set_2EGSPEC \\ & \quad A_27b\ A_27a)\ (\lambda V4x \in A_27a. (ap\ (ap\ (c_2Epair_2E_2C\ A_27b\ 2) \\ & \quad (ap\ V0f\ V4x))\ (ap\ V3P\ V4x)))))))))) \\ & \hspace{15em} (21) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0S \in (2^{A_27a}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ \\ & \quad V0S)) \Rightarrow (\forall V1t \in (2^{A_27b}). (\forall V2f \in (A_27b^{A_27a}). ((\\ & \quad (p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EBIJ\ A_27a\ A_27b)\ V2f)\ V0S)\ V1t)) \wedge \\ & \quad (p\ (ap\ (c_2Epred_set_2EFINITE\ A_27b)\ V1t))) \Rightarrow ((ap\ (c_2Epred_set_2ECARD \\ & \quad A_27a)\ V0S) = (ap\ (c_2Epred_set_2ECARD\ A_27b)\ V1t)))))) \\ & \hspace{15em} (22) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Enum_2Enum)\ V0m)\ (ap\ c_2Epred_set_2Ecount \\ & \quad V1n))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0m)\ V1n)))) \\ & \hspace{15em} (23) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. (p\ (ap\ (c_2Epred_set_2EFINITE \\ & \quad ty_2Enum_2Enum)\ (ap\ c_2Epred_set_2Ecount\ V0n)))) \\ & \hspace{15em} (24) \end{aligned}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. ((ap (c_2Epred_set_2ECARD ty_2Enum_2Enum) (ap c_2Epred_set_2Ecount V0n)) = V0n)) \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((p (ap \\ & (c_2Epred_set_2EFINITE A_27a) V0s)) \Rightarrow (\exists V1f \in (A_27a^{ty_2Enum_2Enum}). \\ & (\exists V2b \in ty_2Enum_2Enum. (p (ap (ap (ap (c_2Epred_set_2EBIJ \\ & ty_2Enum_2Enum A_27a) V1f) (ap c_2Epred_set_2Ecount V2b)) V0s)))))) \end{aligned} \quad (26)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((p (ap \\ & (c_2Epred_set_2EFINITE A_27a) V0s)) \Rightarrow (\exists V1f \in (A_27a^{ty_2Enum_2Enum}). \\ & ((\forall V2n \in ty_2Enum_2Enum. (\forall V3m \in ty_2Enum_2Enum. \\ & (((p (ap (ap c_2Eprim_rec_2E_3C V2n) (ap (c_2Epred_set_2ECARD \\ & A_27a) V0s))) \wedge (p (ap (ap c_2Eprim_rec_2E_3C V3m) (ap (c_2Epred_set_2ECARD \\ & A_27a) V0s)))) \Rightarrow (((ap V1f V2n) = (ap V1f V3m)) \Rightarrow (V2n = V3m)))))) \wedge (V0s = \\ & (ap (c_2Epred_set_2EGSPEC A_27a ty_2Enum_2Enum) (\lambda V4n \in ty_2Enum_2Enum. \\ & (ap (ap (c_2Epair_2E_2C A_27a 2) (ap V1f V4n)) (ap (ap c_2Eprim_rec_2E_3C \\ & V4n) (ap (c_2Epred_set_2ECARD A_27a) V0s)))))))))) \end{aligned}$$