

thm_2Epred_set_EFUNSET_DFUNSET
(TMF4uP3yFEzYQrZH2CnRdnCjEV7WMD9WzQC)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Definition 6 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 7 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A_27b}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V2P \in 2.V2P)) (\lambda V3Q \in 2.V3Q)))$

Definition 9 We define $c_2Epred_set_2EDFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A_27b}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V2P \in 2.V2P)) (\lambda V3Q \in 2.V3Q)))$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V1x)))) \tag{2}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{3}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{4}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (5)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in 2. (((((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27))))))) \quad (6)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27b. ((ap (ap (c.2Ecombin_2EK A.27a A.27b) V0x) V1y) = V0x))) \quad (7)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in (2^{A.27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a. ((p (ap (ap (c.2Ebool_2EIN A.27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c.2Ebool_2EIN A.27a) V2x) V1t))))))) \quad (8)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0f \in (A.27b^{A.27a}). (\forall V1P \in (2^{A.27a}). (\forall V2Q \in (2^{A.27b}). ((p (ap (ap (c.2Ebool_2EIN (A.27b^{A.27a})) V0f) (ap (ap (c.2Epred_set_2EFUNSET A.27a A.27b) V1P) V2Q))) \Leftrightarrow (\forall V3x \in A.27a. ((p (ap (ap (c.2Ebool_2EIN A.27a) V3x) V1P)) \Rightarrow (p (ap (ap (c.2Ebool_2EIN A.27b) (ap V0f V3x)) V2Q)))))))))) \quad (9)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0f \in (A.27b^{A.27a}). (\forall V1P \in (2^{A.27a}). (\forall V2Q \in ((2^{A.27b})^{A.27a}). ((p (ap (ap (c.2Ebool_2EIN (A.27b^{A.27a})) V0f) (ap (ap (c.2Epred_set_2EDFUNSET A.27a A.27b) V1P) V2Q))) \Leftrightarrow (\forall V3x \in A.27a. ((p (ap (ap (c.2Ebool_2EIN A.27a) V3x) V1P)) \Rightarrow (p (ap (ap (c.2Ebool_2EIN A.27b) (ap V0f V3x)) (ap V2Q V3x)))))))))) \quad (10)$$

Theorem 1

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0x \in (2^{A.27a}). (\forall V1y \in (2^{A.27b}). ((ap (ap (c.2Epred_set_2EFUNSET A.27a A.27b) V0x) V1y) = (ap (ap (c.2Epred_set_2EDFUNSET A.27a A.27b) V0x) (ap (c.2Ecombin_2EK (2^{A.27b}) A.27a) V1y))))))$$