

thm\_2Epred\_\_set\_2EFUNSET\_\_applied  
(TMUhgH-  
sWQ57CxYdmPRe28SbRDXj2FVushro)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$   
of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Emarker\_2ECong$  to be  $\lambda V0x \in 2.V0x$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$   
of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 7** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in (2^{A\_27b}).$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ & \quad \forall V0f \in (A\_27b^{A\_27a}).(\forall V1P \in (2^{A\_27a}).(\forall V2Q \in \\ & \quad (2^{A\_27b}).((p (ap (ap (c\_2Ebool\_2EIN (A\_27b^{A\_27a})) V0f) (ap (ap \\ & \quad (c\_2Epred\_set\_2EFUNSET A\_27a A\_27b) V1P) V2Q)))) \Leftrightarrow (\forall V3x \in \\ & \quad A\_27a.((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V3x) V1P)) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN \\ & \quad A\_27b) (ap V0f V3x)) V2Q))))))))) \end{aligned} \tag{1}$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ & \quad \forall V0f \in (A\_27b^{A\_27a}).(\forall V1P \in (2^{A\_27a}).(\forall V2Q \in \\ & \quad (2^{A\_27b}).((p (ap (ap (ap (c\_2Epred\_set\_2EFUNSET A\_27a A\_27b) \\ & \quad V1P) V2Q) V0f)) \Leftrightarrow (\forall V3x \in A\_27a.((p (ap (ap (c\_2Ebool\_2EIN \\ & \quad A\_27a) V3x) V1P)) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN A\_27b) (ap V0f V3x)) \\ & \quad V2Q))))))))) \end{aligned}$$