

# thm\_2Epred\_set\_2EINJ\_COMPOSE (TMJTfi- HADY46jYmLWBLimXUeNxQppRaLGVA)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2ET` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define `c_2Ebool_2E_21` to be  $\lambda A. \lambda 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x)))$

**Definition 4** We define `c_2Ecombin_2Eo` to be  $\lambda A. \lambda 27a : \iota. \lambda A. \lambda 27b : \iota. \lambda A. \lambda 27c : \iota. \lambda V0f \in (A. 27b^{A-27c}). \lambda V1g \in (A. 27c^{A-27b}).$

**Definition 5** We define `c_2Ebool_2EIN` to be  $\lambda A. \lambda 27a : \iota. (\lambda V0x \in A. 27a. (\lambda V1f \in (2^{A-27a}). (ap V1f V0x)))$

**Definition 6** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 7** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

**Definition 8** We define `c_2Epred_set_2EINJ` to be  $\lambda A. \lambda 27a : \iota. \lambda A. \lambda 27b : \iota. \lambda V0f \in (A. 27b^{A-27a}). \lambda V1s \in (2^{A-27b}).$

Assume the following.

$$\begin{aligned}
 & \forall A. 27a. nonempty A. 27a \Rightarrow \forall A. 27b. nonempty A. 27b \Rightarrow \forall A. 27c. \\
 & nonempty A. 27c \Rightarrow (\forall V0f \in (A. 27b^{A-27a}). (\forall V1g \in (A. 27c^{A-27b}). \\
 & (\forall V2x \in A. 27c. ((ap (ap (ap (c_2Ecombin_2Eo A. 27c A. 27b A. 27a) \\
 & V0f) V1g) V2x) = (ap V0f (ap V1g V2x))))))
 \end{aligned} \tag{1}$$

**Theorem 1**

$$\begin{aligned}
 & \forall A. 27a. nonempty A. 27a \Rightarrow \forall A. 27b. nonempty A. 27b \Rightarrow \forall A. 27c. \\
 & nonempty A. 27c \Rightarrow (\forall V0f \in (A. 27b^{A-27a}). (\forall V1g \in (A. 27c^{A-27b}). \\
 & (\forall V2s \in (2^{A-27a}). (\forall V3t \in (2^{A-27b}). (\forall V4u \in \\
 & (2^{A-27c}). ((p (ap (ap (ap (c_2Epred_set_2EINJ A. 27a A. 27b) V0f) \\
 & V2s) V3t)) \wedge (p (ap (ap (ap (c_2Epred_set_2EINJ A. 27b A. 27c) V1g) \\
 & V3t) V4u)))) \Rightarrow (p (ap (ap (ap (c_2Epred_set_2EINJ A. 27a A. 27c) (ap \\
 & (ap (c_2Ecombin_2Eo A. 27a A. 27c A. 27b) V1g) V0f)) V2s) V4u))))))
 \end{aligned}$$