

# thm\_2Epred\_set\_2EINSERT\_DIFF (TMZXY- DMQ1SRxpwMUN9W8cNsZoUK56k92Bwu)

October 26, 2020

**Definition 1** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define `c_2Ebool_2ET` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A_{.27a} : \iota.(\lambda V0P \in (2^{A_{.27a}}).(ap (ap (c_2Emin_2E_3D (2^{A_{.27a}}))$

**Definition 5** We define `c_2Ebool_2EF` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 6** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 7** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define `c_2Ebool_2ECOND` to be  $\lambda A_{.27a} : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_{.27a}.(\lambda V2t2 \in A_{.27a}.(ap (c_2Emin_2E_40 (2^{A_{.27a}}))$

**Definition 9** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

**Definition 10** We define `c_2Ebool_2EIN` to be  $\lambda A_{.27a} : \iota.(\lambda V0x \in A_{.27a}.(\lambda V1f \in (2^{A_{.27a}}).(ap V1f V0x)))$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

Let `c_2Epair_2EABS_prod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow c\_2Epair\_2EABS\_prod A_{.27a} A_{.27b} \in ((ty\_2Epair\_2Eprod A_{.27a} A_{.27b})^{((2^{A_{.27b}})^{A_{.27a}})}) \quad (2)$$

**Definition 11** We define `c_2Epair_2E_2C` to be  $\lambda A_{.27a} : \iota.\lambda A_{.27b} : \iota.\lambda V0x \in A_{.27a}.\lambda V1y \in A_{.27b}.(ap (c_2Emin_2E_40 (2^{A_{.27a}}))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \end{aligned} \quad (3)$$

**Definition 12** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_2Epred\_set\_2EGSPEC\ V0s)\ V1t)$

**Definition 13** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (V0t1\ V1t2))))$

**Definition 14** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A\_27a}). (ap\ (c\_2Epred\_set\_2EGSPEC\ V0x)\ V1s)$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (5)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee (\neg(p\ V0t)))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (8)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\ (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (11)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (12)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\
& p \ V0t))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\
& A\_27a. (((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2ET) \ V0t1) \\
& V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2EF) \\
& V0t1) \ V1t2) = V1t2))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\
& (2^{A\_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a. ((p \ (ap \ (ap \ (c\_2Ebool\_2EIN \\
& A\_27a) \ V2x) \ V0s)) \Leftrightarrow (p \ (ap \ (ap \ (c\_2Ebool\_2EIN \ A\_27a) \ V2x) \ V1t))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\
& (2^{A\_27a}). (\forall V2x \in A\_27a. ((p \ (ap \ (ap \ (c\_2Ebool\_2EIN \ A\_27a) \\
& V2x) \ (ap \ (ap \ (c\_2Epred\_set\_2EDIFF \ A\_27a) \ V0s) \ V1t))) \Leftrightarrow ((p \ (ap \ ( \\
& ap \ (c\_2Ebool\_2EIN \ A\_27a) \ V2x) \ V0s)) \wedge (\neg(p \ (ap \ (ap \ (c\_2Ebool\_2EIN \\
& A\_27a) \ V2x) \ V1t))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\
& A\_27a. (\forall V2s \in (2^{A\_27a}). ((p \ (ap \ (ap \ (c\_2Ebool\_2EIN \ A\_27a) \\
& V0x) \ (ap \ (ap \ (c\_2Epred\_set\_2EINSERT \ A\_27a) \ V1y) \ V2s))) \Leftrightarrow ((V0x = \\
& V1y) \vee (p \ (ap \ (ap \ (c\_2Ebool\_2EIN \ A\_27a) \ V0x) \ V2s))))))
\end{aligned} \tag{17}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\
& (2^{A\_27a}). (\forall V2x \in A\_27a. ((ap \ (ap \ (c\_2Epred\_set\_2EDIFF \\
& A\_27a) \ (ap \ (ap \ (c\_2Epred\_set\_2EINSERT \ A\_27a) \ V2x) \ V0s)) \ V1t) = \\
& (ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ (2^{A\_27a})) \ (ap \ (ap \ (c\_2Ebool\_2EIN \\
& A\_27a) \ V2x) \ V1t)) \ (ap \ (ap \ (c\_2Epred\_set\_2EDIFF \ A\_27a) \ V0s) \ V1t)) \\
& (ap \ (ap \ (c\_2Epred\_set\_2EINSERT \ A\_27a) \ V2x) \ (ap \ (ap \ (c\_2Epred\_set\_2EDIFF \\
& A\_27a) \ V0s) \ V1t))))))
\end{aligned}$$