

thm\_2Epred\_\_set\_2EIN\_\_INSERT\_\_EXPAND  
 (TMaBiVn-  
 LoGvE71MC2NHFscbiVge1Tsmy5pY)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A.\lambda a : \iota.(\lambda V0x \in A.\lambda a.(2^{A-27a}).(ap V1f V0x))$

**Definition 4** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 6** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.\lambda a.\lambda b \in A. (c\_2Epair\_2EABS\_prod A a b) \in ((ty\_2Epair\_2Eprod A a b)^{(2^{A-27b})^{A-27a}}) \tag{2}$$

**Definition 8** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda V0x \in A.\lambda V1y \in A.\lambda V2z \in A.(ap (c\_2Epair\_2EABS\_prod A a b) (V0x V1y V2z))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.\lambda a.\lambda b \in A. (c\_2Epred\_set\_2EGSPEC A a b) \in ((2^{A-27a})^{(ty\_2Epair\_2Eprod A a b)^{(2^{A-27b})^{A-27a}}}) \tag{3}$$

**Definition 9** We define `c_2Epred_set_2EINSERT` to be  $\lambda A_{.27a} : \iota. \lambda V0x \in A_{.27a}. \lambda V1s \in (2^{A_{.27a}}). (ap (c_{.2}$

**Definition 10** We define `c_2Ebool_2EF` to be  $(ap (c_{.2Ebool_2E.21} 2) (\lambda V0t \in 2.V0t))$ .

**Definition 11** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap (c_{.2Emin_2E.3D_3D_3E} V0t) c_{.2Ebool_2E}$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{5}$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \tag{6}$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{7}$$

Assume the following.

$$\forall A_{.27a}. nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. (V0x = V0x)) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \tag{9}$$

Assume the following.

$$\forall A_{.27a}. nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. (\forall V1y \in A_{.27a}. (\forall V2s \in (2^{A_{.27a}}). ((p (ap (ap (c_{.2Ebool_2EIN} A_{.27a} V0x) (ap (ap (c_{.2Epred_set_2EINSERT} A_{.27a} V1y) V2s))) \Leftrightarrow ((V0x = V1y) \vee (p (ap (ap (c_{.2Ebool_2EIN} A_{.27a} V0x) V2s)))))))))) \tag{10}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{11}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{12}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (13)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (14)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (15)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (16)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (17)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (18)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (19)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (20)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (21)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (22)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (23)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (24)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (25)$$

**Theorem 1**

$$\begin{aligned} & \forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.(\forall V1y \in \\ & A_{27a}.(\forall V2P \in (2^{A_{27a}}).((p (ap (ap (c_{2Ebool\_2EIN} A_{27a}) \\ & V0x) (ap (ap (c_{2Epred\_set\_2EINSERT} A_{27a}) V1y) V2P))) \Leftrightarrow ((V0x = \\ & V1y) \vee ((\neg(V0x = V1y)) \wedge (p (ap (ap (c_{2Ebool\_2EIN} A_{27a}) V0x) V2P)))))))))) \end{aligned}$$