

thm\_2Epred\_set\_2ENUM\_SET\_WOP (TM-FrarP8cEEJUMgUs4GdeBCTdPLXkkxqPPD)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A. \exists a : \iota. (\lambda V0x \in A. \exists a. (\lambda V1f \in (2^{A-27a}). (ap V1f V0x)))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. \exists a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))))$

**Definition 5** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 6** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p \Rightarrow p Q)$  of type  $\iota$ .

**Definition 7** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF))$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. (ap (c\_2Ebool\_2E\_7E V0t1) c\_2Ebool\_2EF))))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (2)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^\omega) \quad (3)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^\omega) \quad (4)$$

**Definition 9** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num (c\_2Enum\_2EREP\_num m))$

**Definition 10** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\lambda x. x \in A \wedge P x) \text{ of type } \iota \Rightarrow \iota.$

**Definition 11** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A. \lambda t : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c\_2Emin\_2E\_40 V0P) V1P)))$

**Definition 12** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (\lambda V2t \in ty\_2Enum\_2Enum. (V2t \in V0m \wedge c\_2Ebool\_2E\_3F V0m V1n)))$

**Definition 13** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (V1t2 \in V0t1))))$

**Definition 14** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (\lambda V2t \in ty\_2Enum\_2Enum. (V2t \in V0m \wedge c\_2Ebool\_2E\_5C\_2F V0m V1n)))$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((\neg(p (ap (ap (c\_2Eprim\_rec\_2E\_3C V0m) V1n)))) \Leftrightarrow (p (ap (ap (c\_2Earithmetic\_2E\_3C\_3D V1n) V0m))))))) \quad (5)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}). ((\exists V1n \in ty\_2Enum\_2Enum. ((p (ap V0P V1n)))) \Rightarrow (\exists V2n \in ty\_2Enum\_2Enum. ((p (ap V0P V2n)) \wedge (\forall V3m \in ty\_2Enum\_2Enum. ((p (ap (ap (c\_2Eprim\_rec\_2E\_3C V3m) V2n)))) \Rightarrow (\neg(p (ap V0P V3m)))))))) \quad (6)$$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (8)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (9)$$

Assume the following.

$$\forall A. \lambda t : \iota. nonempty A \Rightarrow (\forall V0x \in A. \forall V1y \in A. (V0x = V1y) \Leftrightarrow (V1y = V0x))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (11)$$

### Theorem 1

$$(\forall V0s \in (2^{ty\_2Enum\_2Enum}). ((\exists V1n \in ty\_2Enum\_2Enum. ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Enum\_2Enum) V1n) V0s)))) \Leftrightarrow (\exists V2n \in ty\_2Enum\_2Enum. ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Enum\_2Enum) V2n) V0s)) \wedge (\forall V3m \in ty\_2Enum\_2Enum. ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Enum\_2Enum) V3m) V0s)) \Rightarrow (p (ap (ap (c\_2Earithmetic\_2E\_3C\_3D V2n) V3m)))))))))))$$