

thm\_2Epred\_\_set\_2EPROD\_\_SET\_\_THM  
(TMRizMnoW3YncPx7852GbLVekfyRsqqMzTq)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a}))\ (\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))\ (\lambda V2t \in 2.V2t))\ (\lambda V2t \in 2.V2t))$

**Definition 6** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))\ (\lambda V2t \in 2.V2t))$

**Definition 7** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 8** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF))$

**Definition 9** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 10** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))\ (\lambda V2t \in 2.V2t))$

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))\ (\lambda V2t \in 2.V2t))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (4)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (5)$$

**Definition 12** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ V0x\ V1y)$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (6)$$

**Definition 13** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27a)\ V0x\ V1s)$

**Definition 14** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27a)\ V0s\ V1t)$

**Definition 15** We define  $c\_2Epred\_set\_2EDELETE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27a)\ V0s\ V1x)$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

**Definition 16** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E\_21\ A\_27a)\ V0s)$

**Definition 17** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (8)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (9)$$

**Definition 18** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ V0m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

**Definition 19** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B)\ V0n)$

**Definition 20** We define  $c\_2Earithmic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 21** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 22** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 23** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a})) A\_27a))$

Let  $c\_2Epred\_set\_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EITSET \\ A\_27a A\_27b \in (((A\_27b^{A\_27b})^{(2^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})}) \end{aligned} \quad (11)$$

**Definition 24** We define  $c\_2Epred\_set\_2EPROD\_IMAGE$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Enum\_2Enum^{A\_27a})$

**Definition 25** We define  $c\_2Epred\_set\_2EPROD\_SET$  to be  $(ap (c\_2Epred\_set\_2EPROD\_IMAGE ty\_2Enum\_2Enum))$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ \forall V2p \in ty\_2Enum\_2Enum.(((ap (ap c\_2Earithmic\_2E\_2A V1n) \\ V0m) = (ap (ap c\_2Earithmic\_2E\_2A V2p) V0m)) \Leftrightarrow ((V0m = c\_2Enum\_2E0) \vee \\ (V1n = V2p)))))) \end{aligned} \quad (12)$$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ (p V0t) \Rightarrow False) \Leftrightarrow \neg (p V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (16)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2) \Rightarrow (p \ V2t3)) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (19)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x_{27})) \wedge ((p \ V1x_{27}) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_{27})))))) \Rightarrow (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_{27}) \Rightarrow (p \ V3y_{27})))))) \quad (20)$$

Assume the following.

$$\forall A_{27a}.nonempty \ A_{27a} \Rightarrow (\forall V0x \in A_{27a}.((ap \ (c_{2Ecombin}_{2EI} \ A_{27a}) \ V0x) = V0x)) \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty \ A_{27a} \Rightarrow (\forall V0f \in (ty_{2Enum}_{2Enum}^{A_{27a}}). \\ & (((ap \ (ap \ (c_{2Epred\_set\_2EPROD\_IMAGE} \ A_{27a}) \ V0f) \ (c_{2Epred\_set\_2EMPTY} \ A_{27a})) = (ap \ c_{2Earithmetic\_2ENUMERAL} \ (ap \ c_{2Earithmetic\_2EBIT1} \ c_{2Earithmetic\_2EZERO}))) \wedge (\forall V1e \in A_{27a}.(\forall V2s \in \\ & (2^{A_{27a}}).((p \ (ap \ (c_{2Epred\_set\_2EFINITE} \ A_{27a}) \ V2s)) \Rightarrow ((ap \ (ap \ (c_{2Epred\_set\_2EPROD\_IMAGE} \ A_{27a}) \ V0f) \ (ap \ (ap \ (c_{2Epred\_set\_2INSERT} \ A_{27a}) \ V1e) \ V2s)) = (ap \ (ap \ c_{2Earithmetic\_2E\_2A} \ (ap \ V0f \ V1e)) \ (ap \ (ap \ (c_{2Epred\_set\_2EPROD\_IMAGE} \ A_{27a}) \ V0f) \ (ap \ (ap \ (c_{2Epred\_set\_2EDELETE} \ A_{27a}) \ V2s) \ V1e)))))))))) \quad (22) \end{aligned}$$

### Theorem 1

$$\begin{aligned} & (((ap \ c_{2Epred\_set\_2EPROD\_SET} \ (c_{2Epred\_set\_2EMPTY} \ ty_{2Enum}_{2Enum})) = \\ & (ap \ c_{2Earithmetic\_2ENUMERAL} \ (ap \ c_{2Earithmetic\_2EBIT1} \ c_{2Earithmetic\_2EZERO}))) \wedge \\ & (\forall V0x \in ty_{2Enum}_{2Enum}.(\forall V1s \in (2^{ty_{2Enum}_{2Enum}}). \\ & ((p \ (ap \ (c_{2Epred\_set\_2EFINITE} \ ty_{2Enum}_{2Enum}) \ V1s)) \Rightarrow ((ap \ c_{2Epred\_set\_2EPROD\_SET} \ (ap \ (ap \ (c_{2Epred\_set\_2INSERT} \ ty_{2Enum}_{2Enum}) \ V0x) \ V1s)) = (ap \ (ap \ c_{2Earithmetic\_2E\_2A} \ V0x) \ (ap \ c_{2Epred\_set\_2EPROD\_SET} \ (ap \ (ap \ (c_{2Epred\_set\_2EDELETE} \ ty_{2Enum}_{2Enum}) \ V1s) \ V0x)))))))))) \end{aligned}$$