

thm_2Epred_set_EPSUBSET_FINITE (TMMh2Y2n7iNFTqFx8EH9wxFyP2Sw1JGKUHX)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_7E` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_{.27a} : \iota.(\lambda V0P \in (2^{A_{.27a}}).(ap (ap (c_2Emin_2E_3D (2^{A_{.27a}}))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 8 We define `c_2Ebool_2E_2I` to be $\lambda A_{.27a} : \iota.(\lambda V0x \in A_{.27a}.(\lambda V1f \in (2^{A_{.27a}}).(ap V1f V0x)))$

Definition 9 We define `c_2Epred_set_ESUBSET` to be $\lambda A_{.27a} : \iota.\lambda V0s \in (2^{A_{.27a}}).\lambda V1t \in (2^{A_{.27a}}).(ap (c_2Emin_2E_3D_3D_3E$

Definition 10 We define `c_2Epred_set_EPSUBSET` to be $\lambda A_{.27a} : \iota.\lambda V0s \in (2^{A_{.27a}}).\lambda V1t \in (2^{A_{.27a}}).(ap (c_2Emin_2E_3D_3D_3E$

Definition 11 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow c_2Epair_2EABS_prod A_{.27a} A_{.27b} \in ((ty_2Epair_2Eprod A_{.27a} A_{.27b})^{(2^{A_{.27b}})^{A_{.27a}}}) \tag{2}$$

Definition 12 We define `c_2Epair_2E_2C` to be $\lambda A_{.27a} : \iota.\lambda A_{.27b} : \iota.\lambda V0x \in A_{.27a}.\lambda V1y \in A_{.27b}.(ap (c_2Emin_2E_3D_3D_3E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (3)$$

Definition 13 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Ebool_2EF$

Definition 14 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF$

Definition 15 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap\ (c_2Ebool_2E21\ (2$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((p\ (ap \\ (c_2Epred_set_2EFINITE\ A_27a)\ V0s)) \Rightarrow (\forall V1t \in (2^{A_27a}). \\ ((p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V1t)\ V0s)) \Rightarrow (p\ (ap\ (c_2Epred_set_2EFINITE \\ A_27a)\ V1t)))))) \end{aligned} \quad (4)$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((p\ (ap \\ (c_2Epred_set_2EFINITE\ A_27a)\ V0s)) \Rightarrow (\forall V1t \in (2^{A_27a}). \\ ((p\ (ap\ (ap\ (c_2Epred_set_2EPSUBSET\ A_27a)\ V1t)\ V0s)) \Rightarrow (p\ (ap\ (\\ c_2Epred_set_2EFINITE\ A_27a)\ V1t)))))) \end{aligned}$$