

thm\_2Epred\_\_set\_2EPSUBSET\_\_MEMBER  
(TML-  
HtF9NVN9trmK273FHSmvHUsEbRQoMUhh)

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**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ (ap } P \ x)) \text{ of type } \iota \Rightarrow \iota.$

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota.$

**Definition 3** We define `c_2Ebool_2E_3F` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap } (c_2Emin_2E_40 \ A \ V0P))$

**Definition 4** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \ P \Rightarrow \ p \ Q)$  of type  $\iota.$

**Definition 5** We define `c_2Ebool_2E_T` to be  $(\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2. V0x)) \ (\lambda V1x \in 2. V1x))$

**Definition 6** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^{A-27a}) \ V0P))$

**Definition 7** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (c_2Ebool_2E_21 \ 2) \ (\lambda V2t \in 2. V2t))$

**Definition 8** We define `c_2Ebool_2E_F` to be  $(\text{ap } (c_2Ebool_2E_21 \ 2) \ (\lambda V0t \in 2. V0t)).$

**Definition 9** We define `c_2Ebool_2E_IN` to be  $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f \ V0x)))$

**Definition 10** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2. (\text{ap } (\text{ap } c_2Emin_2E_3D_3D_3E \ V0t) \ c_2Ebool_2E_F))$

**Definition 11** We define `c_2Epred__set_2ESUBSET` to be  $\lambda A. 27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (\text{ap } (c_2Ebool_2E_7E \ (2^{A-27a}) \ V0s))$

**Definition 12** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (c_2Ebool_2E_21 \ 2) \ (\lambda V2t \in 2. V2t))$

**Definition 13** We define `c_2Epred__set_2EPSUBSET` to be  $\lambda A. 27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (\text{ap } (c_2Ebool_2E_2F_5C \ (2^{A-27a}) \ V0s))$

Assume the following.

$$\text{True} \tag{1}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \tag{2}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (3)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (4)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (6)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & p V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\ & (2^{A\_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a. ((p (ap (ap (c\_2Ebool\_2EIN \\ & A\_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) V1t))))))) \end{aligned} \quad (9)$$

### Theorem 1

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\ & (2^{A\_27a}). ((p (ap (ap (c\_2Epred\_set\_2EPSUBSET A\_27a) V0s) V1t)) \Leftrightarrow \\ & ((p (ap (ap (c\_2Epred\_set\_2ESUBSET A\_27a) V0s) V1t)) \wedge (\exists V2y \in \\ & A\_27a. ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2y) V1t)) \wedge (\neg(p (ap (ap \\ & (c\_2Ebool\_2EIN A\_27a) V2y) V0s)))))))))) \end{aligned}$$