

thm_2Epred_set_2ESING_IFF_EMPTY_REST
(TMFJSVY-
CdSvZxQUNUeMqw4Meyd4dm9uNGK4)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 11 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 12 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 13 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod A_27a 2)^{A_27b})}) \quad (3)$$

Definition 14 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Ebool_2E_3F A_27a : \iota \Rightarrow \iota \Rightarrow \iota)$

Definition 15 We define $c_2Epred_set_2ESING$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_3F A_27a : \iota \Rightarrow \iota \Rightarrow \iota)$

Let $c_2Epred_set_2ECHOICE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Epred_set_2ECHOICE A_27a \in (A_27a^{(2^{A_27a})}) \quad (4)$$

Definition 16 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Ebool_2E_3F A_27a : \iota \Rightarrow \iota \Rightarrow \iota)$

Definition 17 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1x \in A_27a.(ap (c_2Ebool_2E_3F A_27a : \iota \Rightarrow \iota \Rightarrow \iota)$

Definition 18 We define $c_2Epred_set_2EREST$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (ap (c_2Epred_set_2ECHOICE : \iota \Rightarrow \iota \Rightarrow \iota)$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (10)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in (2^{A_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V1t))))))) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\neg(p (ap (c_2Ebool_2EIN A_27a) V0x) (c_2Epred_set_2EEMPTY A_27a)))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.(\forall V2s \in (2^{A_27a}).((p (ap (ap (c_2Ebool_2EIN A_27a) V0x) (ap (ap (c_2Epred_set_2EINSERT A_27a) V1y) V2s))) \Leftrightarrow ((V0x = V1y) \vee (p (ap (ap (c_2Ebool_2EIN A_27a) V0x) V2s))))))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).((\neg(V0s = (c_2Epred_set_2EEMPTY A_27a))) \Rightarrow ((ap (ap (c_2Epred_set_2EINSERT A_27a) (ap (c_2Epred_set_2ECHOICE A_27a) V0s)) (ap (c_2Epred_set_2EREST A_27a) V0s)) = V0s))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V0x) (ap (ap (c_2Epred_set_2EINSERT A_27a) V1y) (c_2Epred_set_2EEMPTY A_27a)))) \Leftrightarrow (V0x = V1y)))) \quad (18)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. ((ap\ (c.2Epred_set.2ECHOICE\ A.27a)\ (ap\ (ap\ (c.2Epred_set.2EINSERT\ A.27a)\ V0x)\ (c.2Epred_set.2EEMPTY\ A.27a)))) = V0x)) \quad (19)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. ((ap\ (c.2Epred_set.2EREST\ A.27a)\ (ap\ (ap\ (c.2Epred_set.2EINSERT\ A.27a)\ V0x)\ (c.2Epred_set.2EEMPTY\ A.27a)))) = (c.2Epred_set.2EEMPTY\ A.27a))) \quad (20)$$

Theorem 1

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). ((p\ (ap\ (c.2Epred_set.2ESING\ A.27a)\ V0s)) \Leftrightarrow ((\neg(V0s = (c.2Epred_set.2EEMPTY\ A.27a))) \wedge ((ap\ (c.2Epred_set.2EREST\ A.27a)\ V0s) = (c.2Epred_set.2EEMPTY\ A.27a))))))$$