

thm_2Epred__set_2ESUM__IMAGE__DELETE
 (TMatugod-
 cafr2Wm6FQcV2UQ5SgWGBpaKm7a)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ecombin_2EK$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.(\lambda V0x \in A.\lambda V1y \in A.\lambda V0x)$

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0f \in ((A.\lambda c^{A.\lambda b})^{A.\lambda a}))$

Definition 4 We define $c_2Ecombin_2EI$ to be $\lambda A.\lambda a : \iota.(ap (ap (c_2Ecombin_2ES A.\lambda a (A.\lambda a^{A.\lambda a})) A.\lambda a))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \tag{3}$$

Definition 5 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A.\lambda a})).(ap (ap (c_2Emin_2E_3D (2^{A.\lambda a}))$

Definition 7 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 22 We define $c_2\text{Earithmetic_2ENUMERAL}$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2\text{Earithmetic_2E_2B} : \iota$ be given. Assume the following.

$$c_2\text{Earithmetic_2E_2B} \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 23 We define $c_2\text{Earithmetic_2EBIT2}$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2\text{Earithmetic_2E_2B}))$

Definition 24 We define $c_2\text{Earithmetic_2EBIT1}$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2\text{Earithmetic_2E_2B}))$

Definition 25 We define $c_2\text{Earithmetic_2EZERO}$ to be c_2Enum_2E0 .

Definition 26 We define $c_2\text{Epred_set_2EEMPTY}$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 27 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (12)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (13)$$

Definition 28 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (14)$$

Definition 29 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Epair_2EABS_prod))$

Definition 30 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Epair_2EABS_prod))$

Definition 31 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1x \in A_27a.(ap (c_2Epair_2EABS_prod))$

Let $c_2Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EITSET A_27a A_27b \in (((A_27b)^{A_27b})^{(2^{A_27a})})^{((A_27b)^{A_27b})^{A_27a}} \quad (15)$$

Definition 32 We define $c_2Epred_set_2ESUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Enum_2Enum^{A_27a})$

Definition 33 We define $c_2\text{Earithmetic_2E_2C_2D}$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 34 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c_2Ebool_2E_21 (2$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\
& \quad ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\
& \quad V0m) V1n)))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\
& \quad V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))))) \\
& \hspace{15em} (16)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\
& \quad V1n) V0m)))) \\
& \hspace{15em} (17)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) \\
& \quad (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p)))))) \\
& \hspace{15em} (18)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V0n))) \\
& \hspace{15em} (19)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2E_2D \\
& \quad V0m) c_2Enum_2E0) = V0m))) \\
& \hspace{15em} (20)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
& \quad (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
& \quad (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
& \quad (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge (\\
& \quad ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
& \quad (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
& \quad V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
& \quad (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
& \quad V0m) V1n))))))))) \\
& \hspace{15em} (21)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Enum_2Enum. (\forall V1c \in ty_2Enum_2Enum. (\\
& (ap (ap c_2Earithmetic_2E_2D (ap (ap c_2Earithmetic_2E_2B V0a) \\
& V1c)) V1c) = V0a))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& V1n)) V0m))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\
& c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))) V0n))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}). (\forall V1a \in ty_2Enum_2Enum. \\
& (\forall V2b \in ty_2Enum_2Enum. ((p (ap V0P (ap (ap c_2Earithmetic_2E_2D \\
& V1a) V2b))) \Leftrightarrow (\forall V3d \in ty_2Enum_2Enum. (((V2b = (ap (ap c_2Earithmetic_2E_2B \\
& V1a) V3d)) \Rightarrow (p (ap V0P c_2Enum_2E0))) \wedge ((V1a = (ap (ap c_2Earithmetic_2E_2B \\
& V2b) V3d)) \Rightarrow (p (ap V0P V3d))))))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\text{True} \tag{29}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.((p \ V0t) \vee \neg(p \ V0t))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (33)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \wedge ((p \ V1t2) \wedge (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \wedge (p \ V2t3)))))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Rightarrow \neg(p \ V0t))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.(\neg(p \ V0t) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2.(\neg(\neg(p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (43)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (44)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a. (p\ (ap\ V1Q\ V4x))))))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (47)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge (((\neg(p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B)))))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee (p\ V1B)))) \quad (49)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (50)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (51)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}).(\forall V1b \in 2.(\forall V2x \in A_27a. \\ & \quad (\forall V3y \in A_27a.((ap V0f (ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & \quad V1b) V2x) V3y)) = (ap (ap (ap (c_2Ebool_2ECOND A_27b) V1b) (ap V0f \\ & \quad V2x)) (ap V0f V3y))))))))) \end{aligned} \quad (52)$$

Assume the following.

$$(\forall V0b \in 2.(\forall V1t1 \in 2.(\forall V2t2 \in 2.(((p (ap (ap (ap (c_2Ebool_2ECOND 2) V0b) V1t1) V2t2)) \Leftrightarrow (((\neg(p V0b)) \vee (p V1t1)) \wedge ((p V0b) \vee (p V2t2))))))) \quad (53)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ & 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow \\ & ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & \quad (\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a. \\ & \quad (\forall V5y_27 \in A_27a.(((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & \quad ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & \quad V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27 \\ & \quad V5y_27))))))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow ((\forall V0t1 \in A_27a.(\forall V1t2 \in \\ & A_27a.((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & \quad V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a.(\forall V3t2 \in A_27a.((ap \\ & (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V2t1) V3t2) = V3t2)))))) \end{aligned} \quad (56)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((ap (c_2Ecombin_2EI A_27a) V0x) = V0x)) \quad (57)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmic_2E_3C_3D c_2Earithmic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D \\
& (ap c_2Earithmic_2EBIT2 V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m)))))))))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p (ap (ap \\
(c_2Ebool_2EIN A_27a) V0x) (c_2Epred_set_2EEMPTY A_27a)))))) \tag{60}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
& A_27a. (\forall V2s \in (2^{A-27a}). ((p (ap (ap (c_2Ebool_2EIN A_27a) \\
& V0x) (ap (ap (c_2Epred_set_2EINSERT A_27a) V1y) V2s))) \Leftrightarrow ((V0x = \\
& V1y) \vee (p (ap (ap (c_2Ebool_2EIN A_27a) V0x) V2s)))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A-27a}). (\forall V1x \in \\
& A_27a. (\forall V2y \in A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) V1x) \\
& (ap (ap (c_2Epred_set_2EDELETE A_27a) V0s) V2y))) \Leftrightarrow ((p (ap (ap \\
& (c_2Ebool_2EIN A_27a) V1x) V0s)) \wedge (\neg (V1x = V2y)))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1s \in \\
& (2^{A-27a}). ((\neg (p (ap (ap (c_2Ebool_2EIN A_27a) V0x) V1s))) \Leftrightarrow ((ap \\
& (ap (c_2Epred_set_2EDELETE A_27a) V1s) V0x) = V1s))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((ap (ap (c_2Epred_set_2EDELETE \\
& A_27a) (c_2Epred_set_2EEMPTY A_27a)) V0x) = (c_2Epred_set_2EEMPTY \\
& A_27a)))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\
& A.27a. (\forall V2s \in (2^{A.27a}). ((ap\ (ap\ (c.2Epred_set.2EDELETE \\
& A.27a)\ (ap\ (ap\ (c.2Epred_set.2EINSERT\ A.27a)\ V0x)\ V2s))\ V1y) = \\
& (ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ (2^{A.27a}))\ (ap\ (ap\ (c.2Emin.2E_3D \\
& A.27a)\ V0x)\ V1y))\ (ap\ (ap\ (c.2Epred_set.2EDELETE\ A.27a)\ V2s)\ V1y)) \\
& (ap\ (ap\ (c.2Epred_set.2EINSERT\ A.27a)\ V0x)\ (ap\ (ap\ (c.2Epred_set.2EDELETE \\
& A.27a)\ V2s)\ V1y))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(2^{A.27a})}). ((\\
& (p\ (ap\ V0P\ (c.2Epred_set.2EEMPTY\ A.27a))) \wedge (\forall V1s \in (2^{A.27a}). \\
& ((p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ V1s)) \wedge (p\ (ap\ V0P\ V1s))) \Rightarrow \\
& (\forall V2e \in A.27a. ((\neg(p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2e)\ V1s))) \Rightarrow \\
& (p\ (ap\ V0P\ (ap\ (ap\ (c.2Epred_set.2EINSERT\ A.27a)\ V2e)\ V1s)))))) \Rightarrow \\
& (\forall V3s \in (2^{A.27a}). ((p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a) \\
& V3s)) \Rightarrow (p\ (ap\ V0P\ V3s))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1s \in \\
& (2^{A.27a}). ((p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ (ap\ (ap\ (c.2Epred_set.2EDELETE \\
& A.27a)\ V1s)\ V0x))) \Leftrightarrow (p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ V1s))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty.2Enum.2Enum^{A.27a}). \\
& (((ap\ (ap\ (c.2Epred_set.2ESUM_IMAGE\ A.27a)\ V0f)\ (c.2Epred_set.2EEMPTY \\
& A.27a)) = c.2Enum.2E0) \wedge (\forall V1e \in A.27a. (\forall V2s \in (2^{A.27a}). \\
& ((p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ V2s)) \Rightarrow ((ap\ (ap\ (c.2Epred_set.2ESUM_IMAGE \\
& A.27a)\ V0f)\ (ap\ (ap\ (c.2Epred_set.2EINSERT\ A.27a)\ V1e)\ V2s)) = \\
& (ap\ (ap\ c.2Earithmetic.2E.2B\ (ap\ V0f\ V1e))\ (ap\ (ap\ (c.2Epred_set.2ESUM_IMAGE \\
& A.27a)\ V0f)\ (ap\ (ap\ (c.2Epred_set.2EDELETE\ A.27a)\ V2s)\ V1e)))))))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty.2Enum.2Enum^{A.27a}). \\
& (\forall V1s \in (2^{A.27a}). (\forall V2e \in A.27a. ((p\ (ap\ (c.2Epred_set.2EFINITE \\
& A.27a)\ V1s)) \wedge (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2e)\ V1s))) \Rightarrow (p\ (ap \\
& (ap\ c.2Earithmetic.2E.3C.3D\ (ap\ V0f\ V2e))\ (ap\ (ap\ (c.2Epred_set.2ESUM_IMAGE \\
& A.27a)\ V0f)\ V1s))))))
\end{aligned} \tag{69}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{70}$$

Assume the following.

$$(\forall V0A \in 2.((p \ V0A) \Rightarrow ((\neg(p \ V0A)) \Rightarrow False))) \quad (71)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow ((p \ V0A) \Rightarrow False) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \quad (72)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p \ V0A)) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow ((p \ V0A) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \quad (73)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \ V0A)) \Rightarrow False) \Rightarrow (((p \ V0A) \Rightarrow False) \Rightarrow False))) \quad (74)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \Leftrightarrow (p \ V2r)) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))))))) \quad (75)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \wedge (p \ V2r)) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p)))))))) \quad (76)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \vee (p \ V2r)) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))))) \quad (77)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \Rightarrow (p \ V2r)) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((\neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))))) \quad (78)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))) \quad (79)$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0f \in (ty_2Enum_2Enum^{A_{27a}}). \\ & \quad (\forall V1s \in (2^{A_{27a}}). ((p (ap (c_2Epred_set_2EFINITE A_{27a}) \\ & \quad V1s)) \Rightarrow (\forall V2e \in A_{27a}. ((ap (ap (c_2Epred_set_2ESUM_IMAGE \\ & \quad A_{27a} V0f) (ap (ap (c_2Epred_set_2EDELETE A_{27a}) V1s) V2e)) = \\ & \quad (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap (c_2Ebool_2EIN \\ & \quad A_{27a} V2e) V1s)) (ap (ap c_2Earithmetic_2E_2D (ap (ap (c_2Epred_set_2ESUM_IMAGE \\ & \quad A_{27a} V0f) V1s)) (ap V0f V2e))) (ap (ap (c_2Epred_set_2ESUM_IMAGE \\ & \quad A_{27a} V0f) V1s)))))))))) \end{aligned}$$