

thm_2Epred_set_2ESURJ_COMPOSE (TMaeomU5NEBdAfnTGSjo2i1MEfh2TxGs6Lx)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. \lambda 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a})) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (P \Rightarrow Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_27E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)))$

Definition 7 We define `c_2Ecombin_2E_o` to be $\lambda A. \lambda 27a : \iota. \lambda A. \lambda 27b : \iota. \lambda A. \lambda 27c : \iota. \lambda V0f \in (A. 27b^{A-27c}). \lambda V1g \in (A. 27c^{A-27b}). \text{inj_o } (V0f \Rightarrow V1g)$

Definition 8 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P \text{ (ap } P \ x)) \text{ then } (the (\lambda x. x \in A \wedge P \ x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define `c_2Ebool_2E_3F` to be $\lambda A. \lambda 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } V0P) (\lambda V0x \in 2. V0x))))$

Definition 10 We define `c_2Ebool_2E_2IN` to be $\lambda A. \lambda 27a : \iota. (\lambda V0x \in A. 27a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f \ V0x)))$

Definition 11 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Definition 12 We define `c_2Epred_set_2ESURJ` to be $\lambda A. \lambda 27a : \iota. \lambda A. \lambda 27b : \iota. \lambda V0f \in (A. 27b^{A-27a}). \lambda V1s \in (A. 27a^{A-27b}). \text{inj_o } (V0f \Rightarrow V1s)$

Assume the following.

$$\text{True} \tag{1}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((\text{True} \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge \text{True}) \Leftrightarrow \\ & (p \ V0t)) \wedge (((\text{False} \wedge (p \ V0t)) \Leftrightarrow \text{False}) \wedge (((p \ V0t) \wedge \text{False}) \Leftrightarrow \text{False}) \wedge \\ & (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \tag{2}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (3)$$

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$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27a^{A_27c}). \\ & (\forall V2x \in A_27c. ((ap\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a) \\ & V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \end{aligned} \quad (5)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27c^{A_27b}). \\ & (\forall V2s \in (2^{A_27a}). (\forall V3t \in (2^{A_27b}). (\forall V4u \in \\ & (2^{A_27c}). (((p\ (ap\ (ap\ (ap\ (c_2Epred_set_2ESURJ\ A_27a\ A_27b) \\ & V0f)\ V2s)\ V3t)) \wedge (p\ (ap\ (ap\ (ap\ (c_2Epred_set_2ESURJ\ A_27b\ A_27c) \\ & V1g)\ V3t)\ V4u))) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Epred_set_2ESURJ\ A_27a\ A_27c) \\ & (ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27c\ A_27b)\ V1g)\ V0f))\ V2s)\ V4u)))))) \end{aligned}$$