

thm_2Epred_set_2Echooser_def_compute
(TMUmPawQs4eoxZw4Sj1ssZ9MmBmcZLXyokF)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 3 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define c_2Ebool_2E21 to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Emin_2E3D\ (2^2)))$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 6 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT2))$

Definition 7 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 8 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1))$

Definition 9 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $c_2Epred_set_2ECHOICE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Epred_set_2ECHOICE A_27a \in (A_27a^{(2^{A_27a})}) \quad (8)$$

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$.

Definition 12 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 13 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 14 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V2t)))$

Definition 15 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (9)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (10)$$

Definition 16 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_21 2)) (\lambda V2z \in 2.V2z))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (11)$$

Definition 17 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V2t))$

Definition 18 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 19 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Epred_set_2EDIFF A_27a V0s V1t))$

Definition 20 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1x \in A_27a.(ap (c_2EDELETE A_27a V0s V1x))$

Definition 21 We define $c_2Epred_set_2EREST$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (ap (c_2EREST A_27a V0s)))$

Let $c_2Epred_set_2Echooser : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Epred_set_2Echooser A_27a \in ((A_27a^{ty_2Enum_2Enum})^{(2^{A_27a})}) \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0f \in ((A_27a^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}). \\ & \quad (\forall V1g \in (A_27a^{ty_2Enum_2Enum}).((\forall V2n \in ty_2Enum_2Enum. \\ & \quad ((ap V1g (ap c_2Enum_2ESUC V2n)) = (ap (ap V0f V2n) (ap c_2Enum_2ESUC V2n)))) \Leftrightarrow ((\forall V3n \in ty_2Enum_2Enum.((ap V1g (ap c_2Earithmetic_2ENUMERAL \\ & \quad (ap c_2Earithmetic_2EBIT1 V3n))) = (ap (ap V0f (ap (ap c_2Earithmetic_2E_2D \\ & \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V3n))) \\ & \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \\ & \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V3n)))))) \wedge \\ & \quad (\forall V4n \in ty_2Enum_2Enum.((ap V1g (ap c_2Earithmetic_2ENUMERAL \\ & \quad (ap c_2Earithmetic_2EBIT2 V4n))) = (ap (ap V0f (ap c_2Earithmetic_2ENUMERAL \\ & \quad (ap c_2Earithmetic_2EBIT1 V4n))) (ap c_2Earithmetic_2ENUMERAL \\ & \quad (ap c_2Earithmetic_2EBIT2 V4n)))))))))) \end{aligned} \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow ((\forall V0s \in (2^{A_27a}).((ap (\\ & \quad ap (c_2Epred_set_2Echooser A_27a) V0s) c_2Enum_2E0) = (ap (c_2Epred_set_2ECHOICE \\ & \quad A_27a) V0s))) \wedge (\forall V1s \in (2^{A_27a}).(\forall V2n \in ty_2Enum_2Enum. \\ & \quad ((ap (ap (c_2Epred_set_2Echooser A_27a) V1s) (ap c_2Enum_2ESUC \\ & \quad V2n)) = (ap (ap (c_2Epred_set_2Echooser A_27a) (ap (c_2Epred_set_2EREST \\ & \quad A_27a) V1s)) V2n)))))) \end{aligned} \quad (15)$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((\forall V0s \in (2^{A_{27a}}). ((\text{ap } (\\ & \text{ap } (c_{2Epred_set_2Echooser } A_{27a}) V0s) c_{2Enum_2E0}) = (\text{ap } (c_{2Epred_set_2ECHOICE} \\ & A_{27a}) V0s))) \wedge ((\forall V1s \in (2^{A_{27a}}). (\forall V2n \in \text{ty_2Enum_2Enum}. \\ & ((\text{ap } (\text{ap } (c_{2Epred_set_2Echooser } A_{27a}) V1s) (\text{ap } c_{2Earithmetic_2ENUMERAL} \\ & (\text{ap } c_{2Earithmetic_2EBIT1 } V2n))) = (\text{ap } (\text{ap } (c_{2Epred_set_2Echooser} \\ & A_{27a}) (\text{ap } (c_{2Epred_set_2EREST } A_{27a}) V1s)) (\text{ap } (\text{ap } c_{2Earithmetic_2E_2D} \\ & (\text{ap } c_{2Earithmetic_2ENUMERAL} (\text{ap } c_{2Earithmetic_2EBIT1 } V2n)))) \\ & (\text{ap } c_{2Earithmetic_2ENUMERAL} (\text{ap } c_{2Earithmetic_2EBIT1 } c_{2Earithmetic_2EZERO})))))) \wedge \\ & (\forall V3s \in (2^{A_{27a}}). (\forall V4n \in \text{ty_2Enum_2Enum}. ((\text{ap } (\text{ap } \\ & (c_{2Epred_set_2Echooser } A_{27a}) V3s) (\text{ap } c_{2Earithmetic_2ENUMERAL} \\ & (\text{ap } c_{2Earithmetic_2EBIT2 } V4n))) = (\text{ap } (\text{ap } (c_{2Epred_set_2Echooser} \\ & A_{27a}) (\text{ap } (c_{2Epred_set_2EREST } A_{27a}) V3s)) (\text{ap } c_{2Earithmetic_2ENUMERAL} \\ & (\text{ap } c_{2Earithmetic_2EBIT1 } V4n))))))))) \end{aligned}$$