thm_2Epred_set_2Ecountable_INSERT (TMX-iVV4BRwo9TBA2VykeXB2xJe8Zy9CGTku)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap$

Definition 9 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{3}$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (5)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (6)$$

Definition 10 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ $c_2Emin_2E_40

Definition 12 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 13 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 14 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 15 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 16 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (7)$$

Definition 17 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

Definition 18 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ ($c_2Ebool_2E

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(8)$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(9)$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(10)$$

Definition 19 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 20 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(11)$$

Definition 21 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 22 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 23 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Definition 24 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 25 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod$$
$$A0\ A1)$$

$$(12)$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod$$
$$A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})})$$

$$(13)$$

Definition 26 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC$$
$$A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})})$$

$$(14)$$

Definition 27 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_2$

Definition 28 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c_2Ebool_2ET)$.

Definition 29 *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A}$

Definition 30 *We define* c_2Epred__set_2Ecountable *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E\_3F$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = c\_2Enum\_2E0) \Leftrightarrow ((V0m =$$
$$c\_2Enum\_2E0) \wedge (V1n = c\_2Enum\_2E0)))))$$

$$(15)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)$$
$$V2p) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)) \Leftrightarrow (V0m = V1n)))))$$

$$(16)$$

Assume the following.

$$True \qquad (17)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1){\Rightarrow}(p\ V1t2)){\Rightarrow}(((p\ V1t2){\Rightarrow}(p\ V0t1)){\Rightarrow}((p\ V0t1){\Leftrightarrow}(p\ V1t2)))))) \qquad (18)$$

Assume the following.

$$(\forall V0t \in 2.(False{\Rightarrow}(p\ V0t))) \qquad (19)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t){\vee}(\neg(p\ V0t)))) \qquad (20)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)){\Leftrightarrow}(p\ V0t))) \qquad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((True{\wedge}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\wedge}True){\Leftrightarrow}(p\ V0t)){\wedge}(((False{\wedge}(p\ V0t)){\Leftrightarrow}False){\wedge}((((p\ V0t){\wedge}False){\Leftrightarrow}False){\wedge}(((p\ V0t){\wedge}(p\ V0t)){\Leftrightarrow}(p\ V0t))))))) \qquad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True{\vee}(p\ V0t)){\Leftrightarrow}True){\wedge}((((p\ V0t){\vee}True){\Leftrightarrow}True){\wedge}(((False{\vee}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\vee}False){\Leftrightarrow}(p\ V0t)){\wedge}(((p\ V0t){\vee}(p\ V0t)){\Leftrightarrow}(p\ V0t))))))) \qquad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Rightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\Rightarrow}True){\Leftrightarrow}True){\wedge}(((False{\Rightarrow}(p\ V0t)){\Leftrightarrow}True){\wedge}((((p\ V0t){\Rightarrow}(p\ V0t)){\Leftrightarrow}True){\wedge}(((p\ V0t){\Rightarrow}False){\Leftrightarrow}(\neg(p\ V0t)))))))) \qquad (24)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.((V0x = V0x){\Leftrightarrow}True)) \qquad (25)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y){\Leftrightarrow}(V1y = V0x)))) \qquad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\Leftrightarrow}True){\Leftrightarrow}(p\ V0t)){\wedge}(((False{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(\neg(p\ V0t))){\wedge}(((p\ V0t){\Leftrightarrow}False){\Leftrightarrow}(\neg(p\ V0t))))))) \qquad (27)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A)\wedge($$
$$(p\ V1B)\vee(p\ V2C)))\Leftrightarrow(((p\ V0A)\wedge(p\ V1B))\vee((p\ V0A)\wedge(p\ V2C))))))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B)\vee$$
$$(p\ V2C))\wedge(p\ V0A))\Leftrightarrow(((p\ V1B)\wedge(p\ V0A))\vee((p\ V2C)\wedge(p\ V0A))))))) \quad (29)$$

Assume the following.

$$(\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2R \in 2.((((p\ V0P)\vee$$
$$(p\ V1Q))\Rightarrow(p\ V2R))\Leftrightarrow(((p\ V0P)\Rightarrow(p\ V2R))\wedge((p\ V1Q)\Rightarrow(p\ V2R)))))))$$
$$(30)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow$$
$$((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3))))))) \quad (31)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in$$
$$2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow \quad (32)$$
$$(((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in 2.(\forall V1Q \in 2.$$
$$(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.$$
$$(\forall V5y\_27 \in A\_27a.((((p\ V0P)\Leftrightarrow(p\ V1Q))\wedge(((p\ V1Q)\Rightarrow(V2x = V3x\_27))\wedge$$
$$((\neg(p\ V1Q))\Rightarrow(V4y = V5y\_27))))\Rightarrow((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)$$
$$V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)$$
$$V5y\_27)))))))))$$
$$(33)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow((\forall V0t1 \in A\_27a.(\forall V1t2 \in$$
$$A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)$$
$$V1t2) = V0t1)))\wedge(\forall V2t1 \in A\_27a.(\forall V3t2 \in A\_27a.((ap$$
$$(ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V2t1)\ V3t2) = V3t2))))$$
$$(34)$$

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Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$\textsf{c\_2Enum\_2E0})\ V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \textsf{c\_2Enum\_2E0}) = V1n))\wedge((\forall V2n \in$$
$$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V3m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Enumeral\_2EiZ}\ (ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.$$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \textsf{c\_2Enum\_2E0})\ V4n) = \textsf{c\_2Enum\_2E0}))\wedge$$
$$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V5n)\ \textsf{c\_2Enum\_2E0}) = \textsf{c\_2Enum\_2E0}))\wedge((\forall V6n \in ty\_2Enum\_2Enum.$$
$$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($$
$$ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V7m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$\textsf{c\_2Enum\_2E0})\ V8n) = \textsf{c\_2Enum\_2E0}))\wedge((\forall V9n \in ty\_2Enum\_2Enum.$$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \textsf{c\_2Enum\_2E0}) = V9n))\wedge((\forall V10n \in$$
$$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V11m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$$
$$\textsf{c\_2Enum\_2E0})\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$$
$$V12n))) = \textsf{c\_2Enum\_2E0}))\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap$$
$$(ap\ c\_2Earithmetic\_2EEXP\ \textsf{c\_2Enum\_2E0})\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \textsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \textsf{c\_2Enum\_2E0}))\wedge((\forall V14n \in$$
$$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \textsf{c\_2Enum\_2E0}) =$$
$$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO}))))\wedge$$
$$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$$
$$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V15n))$$
$$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap\ \textsf{c\_2Enum\_2ESUC}$$
$$\textsf{c\_2Enum\_2E0}) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$$
$$\textsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V17n \in ty\_2Enum\_2Enum.($$
$$(ap\ \textsf{c\_2Enum\_2ESUC}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \textsf{c\_2Enum\_2ESUC}\ V17n))))\wedge(((ap\ \textsf{c\_2Eprim\_\_rec\_2EPRE}\ \textsf{c\_2Enum\_2E0}) =$$
$$\textsf{c\_2Enum\_2E0})\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \textsf{c\_2Eprim\_\_rec\_2EPRE}$$
$$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \textsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum.$$
$$(((ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \textsf{c\_2Enum\_2E0})\Leftrightarrow(V19n = \textsf{c\_2Earithmetic\_2EZERO})))\wedge$$
$$((\forall V20n \in ty\_2Enum\_2Enum.((\textsf{c\_2Enum\_2E0} = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V20n))\Leftrightarrow(V20n = \textsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V21n \in ty\_2Enum\_2Enum.$$
$$(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V21n) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge$$
$$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V23n)\ \textsf{c\_2Enum\_2E0}))\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ \textsf{c\_2Enum\_2E0})\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V24n)))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ \textsf{c\_2Earithmetic\_2EZERO})$$
$$V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V25n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3E}$$
$$\textsf{c\_2Enum\_2E0})\ V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V28n))\ \textsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ \textsf{c\_2Earithmetic\_2EZERO})$$
$$V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V29n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$\textsf{c\_2Enum\_2E0})\ V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\
((\mathsf{c\_2Earithmetic\_2EZERO} = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)){\Leftrightarrow}False){\wedge}\\
((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO}){\Leftrightarrow}\\
False){\wedge}(((\mathsf{c\_2Earithmetic\_2EZERO} = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\\
V0n)){\Leftrightarrow}False){\wedge}((((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO}){\Leftrightarrow}\\
False){\wedge}((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\\
V1m)){\Leftrightarrow}False){\wedge}((((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\\
V1m)){\Leftrightarrow}False){\wedge}((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\\
V1m)){\Leftrightarrow}(V0n = V1m)){\wedge}(((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\\
V1m)){\Leftrightarrow}(V0n = V1m)))))))))))$$

(36)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\\
A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_set\_2EUNIV}\ A\_27a))))$$

(37)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1y \in\\
A\_27a.(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\\
V0x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_set\_2EINSERT}\ A\_27a)\ V1y)\ V2s))){\Leftrightarrow}((V0x =\\
V1y){\vee}(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V2s)))))))$$

(38)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1s \in\\
(2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V1s)){\Rightarrow}((ap\ (ap\\
(\mathsf{c\_2Epred\_set\_2EINSERT}\ A\_27a)\ V0x)\ V1s) = V1s))))$$

(39)

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1s \in\\
(2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_set\_2Ecountable}\ A\_27a)\ (ap\ (ap\ (\\
\mathsf{c\_2Epred\_set\_2EINSERT}\ A\_27a)\ V0x)\ V1s))){\Leftrightarrow}(p\ (ap\ (\mathsf{c\_2Epred\_set\_2Ecountable}\\
A\_27a)\ V1s)))))$$