thm_2Epred__set_2Epair__to__num__inv (TMVtfc9oauChdbCJq71oLdzxKfAB2BhTC4e)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 7 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{4}$$

Let $c\_2Enumpair\_2Einvtri0 : \iota$ be given. Assume the following.

$$c\_2Enumpair\_2Einvtri0 \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{5}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{6}$$

Definition 8 *We define* c_2Enumpair_2Einvtri *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (c\_2Epair\_2ESND\ ty$

Let $c\_2Enumpair\_2Etri : \iota$ be given. Assume the following.

$$c\_2Enumpair\_2Etri \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{8}$$

Definition 9 *We define* c_2Enumpair_2Ensnd *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{9}$$

Definition 10 *We define* c_2Enumpair_2Enfst *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{10}$$

Definition 11 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 12 *We define* c_2Epred__set_2Enum__to__pair *to be* $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ ($c_2Epai

Definition 13 *We define* c_2Enumpair_2Enpair *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Epred\_\_set\_2Epair\_\_to\_\_num : \iota$ be given. Assume the following.

$$c\_2Epred\_\_set\_2Epair\_\_to\_\_num \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{11}$$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{14}$$

Assume the following.

$$(\forall V0x \in ty\_2Enum\_2Enum.(\forall V1y \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enumpair\_2Enfst}\ (ap\ (ap\ \mathsf{c\_2Enumpair\_2Enpair}\ V0x)\ V1y)) = V0x))) \tag{15}$$

Assume the following.

$$(\forall V0x \in ty\_2Enum\_2Enum.(\forall V1y \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enumpair\_2Ensnd}\ (ap\ (ap\ \mathsf{c\_2Enumpair\_2Enpair}\ V0x)\ V1y)) = V1y))) \tag{16}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ \mathsf{c\_2Enumpair\_2Enpair}\ (ap\ \mathsf{c\_2Enumpair\_2Enfst}\ V0n))\ (ap\ \mathsf{c\_2Enumpair\_2Ensnd}\ V0n)) = V0n)) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \tag{18}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ c\_2Epred\_\_set\_2Epair\_\_to\_\_num\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V0m)\ V1n)) = (ap\ (ap\ \mathsf{c\_2Enumpair\_2Enpair}\ V0m)\ V1n)))) \tag{19}$$

Theorem 1

$$((\forall V0x \in ty\_2Enum\_2Enum.((ap\ c\_2Epred\_\_set\_2Epair\_\_to\_\_num\ (ap\ \mathsf{c\_2Epred\_\_set\_2Enum\_\_to\_\_pair}\ V0x)) = V0x)) \wedge (\forall V1x \in ty\_2Enum\_2Enum.(\forall V2y \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Epred\_\_set\_2Enum\_\_to\_\_pair}\ (ap\ c\_2Epred\_\_set\_2Epair\_\_to\_\_num\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V1x)\ V2y))) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V1x)\ V2y)))))$$