

thm_2Eprim__rec_2EDC
(TMU2iKJHrbWWNYfsUFS67PNseTZDwdSiPZG)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \mathbf{if} \ (\exists x \in A. p \ (ap \ P \ x)) \ \mathbf{then} \ (the \ (\lambda x. x \in A \wedge p \ x))$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_--o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c.2Ebool.2E.3F` to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c.2Emin.2E.40\ A$

Definition 4 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a} ($

Definition 6 We define `c2Ebool.2EF` to be $(\text{ap } (\text{c2Ebool.2E.21 } 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o \ (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 8 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap \text{c_2Emin_2E_3D_3D_3E } V0t) \text{c_2Ebool_2E_7E}$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $c_Enum_EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty-2Enum-2Enum}) \quad (2)$$

Let $c_Enum_ESUC_REP : \iota$ be given. Assume the following.

$$c2Enum_ESUC_REP \in (\omega^{\omega^{\omega}}) \quad (3)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (4)$$

Definition 9 We define `c_2Enum_2ESUC` to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Let $c_2Enum_2EZZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZZERO_REP \in \omega \tag{5}$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZZERO_REP)$.

Let $c_2Eprim_rec_2ESIMP_REC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eprim_rec_2ESIMP_REC\ A_27a \in ((A_27a^{ty_2Enum_2Enum})^{(A_27a^{A_27a})})^{A_27a} \tag{6}$$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \tag{7}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{8}$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ &(p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ &(((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \tag{9}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{10}$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ &(p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ &p\ V0t)))))) \end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned} &(\forall V0P \in (2^{ty_2Enum_2Enum}).(((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\ &(\forall V1n \in ty_2Enum_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\ &V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p\ (ap\ V0P\ V2n)))))) \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned} &\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1f \in \\ &(A_27a^{A_27a}).(((ap\ (ap\ (ap\ (c_2Eprim_rec_2ESIMP_REC\ A_27a) \\ &V0x)\ V1f)\ c_2Enum_2E0) = V0x) \wedge (\forall V2m \in ty_2Enum_2Enum.((\\ &ap\ (ap\ (ap\ (c_2Eprim_rec_2ESIMP_REC\ A_27a)\ V0x)\ V1f)\ (ap\ c_2Enum_2ESUC \\ &V2m)) = (ap\ V1f\ (ap\ (ap\ (ap\ (c_2Eprim_rec_2ESIMP_REC\ A_27a)\ V0x) \\ &V1f)\ V2m)))))) \end{aligned} \tag{13}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1R \in \\
& ((2^{A_27a})^{A_27a}). (\forall V2a \in A_27a. (((p\ (ap\ V0P\ V2a)) \wedge (\forall V3x \in \\
& A_27a. ((p\ (ap\ V0P\ V3x)) \Rightarrow (\exists V4y \in A_27a. ((p\ (ap\ V0P\ V4y)) \wedge (\\
& p\ (ap\ (ap\ V1R\ V3x)\ V4y)))))) \Rightarrow (\exists V5f \in (A_27a^{ty_2Enum_2Enum}). \\
& (((ap\ V5f\ c_2Enum_2E0) = V2a) \wedge (\forall V6n \in ty_2Enum_2Enum. ((\\
& p\ (ap\ V0P\ (ap\ V5f\ V6n))) \wedge (p\ (ap\ (ap\ V1R\ (ap\ V5f\ V6n))\ (ap\ V5f\ (ap\ c_2Enum_2ESUC \\
& V6n))))))))))
\end{aligned}$$