

thm\_2Eprim\_\_rec\_2ELESS\_\_LEMMA2 (TMSGtF-  
fGv22SXLgNX4ad9Hb6wJdd6RSF1MU)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow p \Rightarrow Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 6** We define  $c\_2Ebool\_2E\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2EF$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 9** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$

**Definition 10** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A) \wedge \text{ of type } \iota \Rightarrow \iota.$

**Definition 11** We define `c_2Ebool_2E_3F` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40$

**Definition 12** We define `c_2Eprim_rec_2E_3C` to be  $\lambda V0m \in \text{ty\_2Enum\_2Enum}. \lambda V1n \in \text{ty\_2Enum\_2Enum}.$

Assume the following.

$$\text{True} \tag{5}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((\text{True} \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow \text{True}) \Leftrightarrow \\ & (p \ V0t)) \wedge (((\text{False} \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow \text{False}) \Leftrightarrow \neg( \\ & p \ V0t)))))) \end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in \text{ty\_2Enum\_2Enum}. (p \text{ (ap (ap c_2Eprim\_rec_2E_3C } V0n) \\ & \text{(ap c_2Enum\_2ESUC } V0n)))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in \text{ty\_2Enum\_2Enum}. (\forall V1n \in \text{ty\_2Enum\_2Enum}. ( \\ & (p \text{ (ap (ap c_2Eprim\_rec_2E_3C } V0m) \ V1n)) \Rightarrow (p \text{ (ap (ap c_2Eprim\_rec_2E_3C} \\ & \ V0m) \text{ (ap c_2Enum\_2ESUC } V1n)))))) \end{aligned} \tag{8}$$

**Theorem 1**

$$\begin{aligned} & (\forall V0m \in \text{ty\_2Enum\_2Enum}. (\forall V1n \in \text{ty\_2Enum\_2Enum}. ( \\ & ((V0m = V1n) \vee (p \text{ (ap (ap c_2Eprim\_rec_2E_3C } V0m) \ V1n))) \Rightarrow (p \text{ (ap (} \\ & \text{ap c_2Eprim\_rec_2E_3C } V0m) \text{ (ap c_2Enum\_2ESUC } V1n)))))) \end{aligned}$$