thm_2Eprim__rec_2EPRIM__REC__THM (TMNFYfCdAT5psLeSNLAMzvqsfSk2q6V8RJ6)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (2)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (3)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (4)$$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (5)$$

Definition 5 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Eprim\_\_rec\_2ESIMP\_\_REC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eprim\_\_rec\_2ESIMP\_\_REC\ A\_27a \in (((A\_27a^{ty\_2Enum\_2Enum})^{(A\_27a^{A\_27a})})^{A\_27a}) \tag{6}$$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 11 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2E}$

Definition 12 *We define* c_2Eprim__rec_2EPRIM__REC__FUN *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1f \in ((A\_2$

Definition 13 *We define* c_2Eprim__rec_2EPRIM__REC *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1f \in ((A\_27a^{ty\_2}$

Assume the following.

$$True \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27b.((ap\ (\lambda V2x \in A\_27b.V0t1)\ V1t2) = V0t1))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{9}$$

Assume the following.

$$(((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}) \wedge (\forall V0m \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m)) = V0m))) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1f \in (A\_27a^{A\_27a}).(((ap\ (ap\ (ap\ (c\_2Eprim\_\_rec\_2ESIMP\_\_REC\ A\_27a)\ V0x)\ V1f)\ \mathsf{c\_2Enum\_2E0}) = V0x) \wedge (\forall V2m \in ty\_2Enum\_2Enum.((ap\ (ap\ (ap\ (c\_2Eprim\_\_rec\_2ESIMP\_\_REC\ A\_27a)\ V0x)\ V1f)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2m)) = (ap\ V1f\ (ap\ (ap\ (ap\ (c\_2Eprim\_\_rec\_2ESIMP\_\_REC\ A\_27a)\ V0x)\ V1f)\ V2m))))))) \tag{11}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1f \in ((A\_27a^{ty\_2Enum\_2Enum})^{A\_27a}).(((ap\ (ap\ (ap\ (\mathsf{c\_2Eprim\_\_rec\_2EPRIM\_\_REC}\ A\_27a)\ V0x)\ V1f)\ \mathsf{c\_2Enum\_2E0}) = V0x) \wedge (\forall V2m \in ty\_2Enum\_2Enum. ((ap\ (ap\ (ap\ (\mathsf{c\_2Eprim\_\_rec\_2EPRIM\_\_REC}\ A\_27a)\ V0x)\ V1f)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2m)) = (ap\ (ap\ V1f\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eprim\_\_rec\_2EPRIM\_\_REC}\ A\_27a)\ V0x)\ V1f)\ V2m))\ V2m))))))$$