thm_2Eprim__rec_2ESIMP__REC__REL__UNIQUE (TMGzeQx3PGTtdFgXHd5XMUe7z6ykWZVVwTw)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{3}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

Definition 7 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

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Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p \ (ap \ P \ x))$ **then** $(the \ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap \ V0P \ (ap \ (\text{c\_2Emin\_2E\_40}$

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (5)$$

Definition 12 *We define* c_2Enum_2E0 *to be* $(ap \ c\_2Enum\_2EABS\_\_num \ c\_2Enum\_2EZERO\_\_REP).$

Definition 13 *We define* c_2Eprim__rec_2ESIMP__REC__REL *to be* $\lambda A\_27a : \iota.\lambda V0fun \in (A\_27a^{ty\_2Enum\_2E}$

Assume the following.

$$True \qquad (6)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((((p \ V0t) \Rightarrow True) \Leftrightarrow$$
$$True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\qquad (7)$$
$$(p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t))))))))$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow$$
$$True)) \qquad (8)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p \ (ap \ V0P \ \text{c\_2Enum\_2E0})) \wedge$$
$$(\forall V1n \in ty\_2Enum\_2Enum.((p \ (ap \ V0P \ V1n)) \Rightarrow (p \ (ap \ V0P \ (ap \ \text{c\_2Enum\_2ESUC}$$
$$V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p \ (ap \ V0P \ V2n)))))$$
$$(9)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(p \ (ap \ (ap \ \text{c\_2Eprim\_\_rec\_2E\_3C} \ (ap \ \text{c\_2Enum\_2ESUC} \ V0m)) \ V1n)) \Rightarrow (\qquad (10)$$
$$p \ (ap \ (ap \ \text{c\_2Eprim\_\_rec\_2E\_3C} \ V0m) \ V1n)))))$$

Theorem 1

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1f \in$$
$$(A\_27a^{A\_27a}).(\forall V2g1 \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V3g2 \in$$
$$(A\_27a^{ty\_2Enum\_2Enum}).(\forall V4m1 \in ty\_2Enum\_2Enum.(\forall V5m2 \in$$
$$ty\_2Enum\_2Enum.(((p \ (ap \ (ap \ (ap \ (ap \ (ap \ (\text{c\_2Eprim\_\_rec\_2ESIMP\_\_REC\_\_REL}$$
$$A\_27a) \ V2g1) \ V0x) \ V1f) \ V4m1)) \wedge (p \ (ap \ (ap \ (ap \ (ap \ (ap \ (\text{c\_2Eprim\_\_rec\_2ESIMP\_\_REC\_\_REL}$$
$$A\_27a) \ V3g2) \ V0x) \ V1f) \ V5m2))) \Rightarrow (\forall V6n \in ty\_2Enum\_2Enum.($$
$$((p \ (ap \ (ap \ \text{c\_2Eprim\_\_rec\_2E\_3C} \ V6n) \ V4m1)) \wedge (p \ (ap \ (ap \ \text{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V6n) \ V5m2))) \Rightarrow ((ap \ V2g1 \ V6n) = (ap \ V3g2 \ V6n)))))))))))$$