thm_2Eprim__rec_2ESIMP__REC__REL__UNIQUE__RESULT (TMH-BuzXjqGq3kXGDKBacbwLMdpPQHCcuLRA)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A$

Definition 10 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ c\_2Ebool\_2E\_2F\_5C$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{3}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

Definition 11 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 12 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{5}$$

Definition 13 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 14 *We define* c_2Eprim__rec_2ESIMP__REC__REL *to be* $\lambda A\_27a : \iota.\lambda V0fun \in (A\_27a^{ty\_2Enum\_2E}$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{7}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F\_21}\ A\_27a)\ (\lambda V1x \in A\_27a.(ap\ V0P\ V1x)))) \Leftrightarrow ((\exists V2x \in A\_27a.(p\ (ap\ V0P\ V2x))) \wedge (\forall V3x \in A\_27a.(\forall V4y \in A\_27a.(((p\ (ap\ V0P\ V3x)) \wedge (p\ (ap\ V0P\ V4y))) \Rightarrow (V3x = V4y))))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a))))) \tag{10}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)))) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1f \in (A\_27a^{A\_27a}).(\forall V2n \in ty\_2Enum\_2Enum.(\exists V3fun \in (A\_27a^{ty\_2Enum\_2Enum}).(p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eprim\_\_rec\_2ESIMP\_\_REC\_\_REL}\ A\_27a)\ V3fun)\ V0x)\ V1f)\ V2n)))))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1f \in (A\_27a^{A\_27a}).(\forall V2g1 \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V3g2 \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V4m1 \in ty\_2Enum\_2Enum.(\forall V5m2 \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eprim\_\_rec\_2ESIMP\_\_REC\_\_REL}\ A\_27a)\ V2g1)\ V0x)\ V1f)\ V4m1)) \land (p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eprim\_\_rec\_2ESIMP\_\_REC\_\_REL}\ A\_27a)\ V3g2)\ V0x)\ V1f)\ V5m2))) \Rightarrow (\forall V6n \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V6n)\ V4m1)) \land (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V6n)\ V5m2))) \Rightarrow ((ap\ V2g1\ V6n) = (ap\ V3g2\ V6n))))))))))) \tag{13}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1f \in (A\_27a^{A\_27a}).(\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F\_21}\ A\_27a)\ (\lambda V3y \in A\_27a.(ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ (A\_27a^{ty\_2Enum\_2Enum}))\ (\lambda V4g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eprim\_\_rec\_2ESIMP\_\_REC\_\_REL}\ A\_27a)\ V4g)\ V0x)\ V1f)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n)))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ V3y)\ (ap\ V4g\ V2n)))))))))))$$