thm_2Eprim__rec_2ESIMP__REC__THM (TMGVhix-pNA9FFuApKLn2fn6ifNTM8EL7AzU)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 7 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Eprim\_\_rec\_2ESIMP\_\_REC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eprim\_\_rec\_2ESIMP\_\_REC\ A\_27a \in (((A\_27a^{ty\_2Enum\_2Enum})^{(A\_27a^{A\_27a})})^{A\_27a}) \tag{4}$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{5}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{6}$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 12 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 13 *We define* c_2Eprim__rec_2ESIMP__REC__REL *to be* $\lambda A\_27a : \iota.\lambda V0fun \in (A\_27a^{ty\_2Enum\_2E}$

Assume the following.

$$True \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1f \in (A\_27a^{A\_27a}).(\forall V2g1 \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V3g2 \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V4m1 \in ty\_2Enum\_2Enum.(\forall V5m2 \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eprim\_\_rec\_2ESIMP\_\_REC\_\_REL}\ A\_27a)\ V2g1)\ V0x)\ V1f)\ V4m1)) \wedge (p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eprim\_\_rec\_2ESIMP\_\_REC\_\_REL}\ A\_27a)\ V3g2)\ V0x)\ V1f)\ V5m2))) \Rightarrow (\forall V6n \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V6n)\ V4m1)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V6n)\ V5m2))) \Rightarrow ((ap\ V2g1\ V6n) = (ap\ V3g2\ V6n)))))))))) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1f\_27 \in (A\_27a^{A\_27a}).(\forall V2n \in ty\_2Enum\_2Enum.(\exists V3g \in (A\_27a^{ty\_2Enum\_2Enum}).((p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eprim\_\_rec\_2ESIMP\_\_REC\_\_REL}\ A\_27a)\ V3g)\ V0x)\ V1f\_27)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n))) \wedge ((ap\ (ap\ (ap\ (c\_2Eprim\_\_rec\_2ESIMP\_\_REC\ A\_27a)\ V0x)\ V1f\_27)\ V2n) = (ap\ V3g\ V2n))))))) \tag{11}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m)))))) \tag{12}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1f \in (A\_27a^{A\_27a}).(((ap\ (ap\ (ap\ (c\_2Eprim\_\_rec\_2ESIMP\_\_REC\ A\_27a)\ V0x)\ V1f)\ \mathsf{c\_2Enum\_2E0}) = V0x) \wedge (\forall V2m \in ty\_2Enum\_2Enum.((ap\ (ap\ (ap\ (c\_2Eprim\_\_rec\_2ESIMP\_\_REC\ A\_27a)\ V0x)\ V1f)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2m)) = (ap\ V1f\ (ap\ (ap\ (ap\ (c\_2Eprim\_\_rec\_2ESIMP\_\_REC\ A\_27a)\ V0x)\ V1f)\ V2m)))))))$$