# thm_2Eprim_rec_2ESUC_LESS (TMG8vJ7sfHR7XMi3N3M1JevUEh14p8Do67n)

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**Definition 1** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\__o\ (x = y)$
*of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

**Definition 3** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27a})$

**Definition 4** *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

**Definition 5** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\__o\ (p\ P \Rightarrow p\ Q)$
*of type* $\iota$.

**Definition 6** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Enum\_2EREP\__num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\__num \in (omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Enum\_2ESUC\__REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\__REP \in (omega^{omega}) \tag{3}$$

Let $c\_2Enum\_2EABS\__num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\__num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

**Definition 7** *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\__num\ ($

**Definition 8** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

**Definition 9** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \wedge p$
*of type* $\iota \Rightarrow \iota$.

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**Definition 10** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\textsf{c\_2Emin\_2E\_40}$

**Definition 11** *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Assume the following.
$$True \tag{5}$$

Assume the following.
$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{6}$$

Assume the following.
$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \tag{7}$$

Assume the following.
$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t))) \tag{8}$$

Assume the following.
$$(\forall V0t \in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow (p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge (((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{9}$$

Assume the following.
$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge ((\neg False)\Leftrightarrow True))) \tag{10}$$

Assume the following.
$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg( p\ V0t))))))) \tag{11}$$

**Theorem 1**
$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( (p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V0m))\ V1n))\Rightarrow( p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n)))))$$