thm_2Eprim__rec_2Enum__Axiom__old (TMRbVLpZ5FhbmLWcjNmrXujfkrnPgzSt44t)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40 A

Definition 4 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 5 *We define* c_2Ebool_2ET *to be* $(ap\ (ap$ (c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ (c_2Emin_2E_3D $(2^{A\_27a}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ c_2Ebool_2E_2F_5C

Definition 9 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (2)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (3)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (4)$$

Definition 10 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{5}$$

Definition 11 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 12 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 13 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ ($c_2Ebool_2E

Let $c\_2Eprim\_\_rec\_2ESIMP\_\_REC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eprim\_\_rec\_2ESIMP\_\_REC\ A\_27a \in (((A\_27a^{ty\_2Enum\_2Enum})^{(A\_27a^{A\_27a})})^{A\_27a}) \tag{6}$$

Definition 14 *We define* c_2Eprim__rec_2EPRIM__REC__FUN *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1f \in ((A\_2$

Definition 15 *We define* c_2Eprim__rec_2EPRIM__REC *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1f \in ((A\_27a^{ty\_2}$

Assume the following.

$$True \tag{7}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{9}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{11}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0})) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n))))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1f \in ((A\_27a^{ty\_2Enum\_2Enum})^{A\_27a}).(((ap\ (ap\ (ap\ (\mathsf{c\_2Eprim\_\_rec\_2EPRIM\_\_REC}\ A\_27a)\ V0x)\ V1f)\ \mathsf{c\_2Enum\_2E0}) = V0x) \wedge (\forall V2m \in ty\_2Enum\_2Enum.((ap\ (ap\ (ap\ (\mathsf{c\_2Eprim\_\_rec\_2EPRIM\_\_REC}\ A\_27a)\ V0x)\ V1f)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2m)) = (ap\ (ap\ V1f\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eprim\_\_rec\_2EPRIM\_\_REC}\ A\_27a)\ V0x)\ V1f)\ V2m))\ V2m)))))) \tag{13}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0e \in A\_27a.(\forall V1f \in ((A\_27a^{ty\_2Enum\_2Enum})^{A\_27a}).(p\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F\_21}\ (A\_27a^{ty\_2Enum\_2Enum}))\ (\lambda V2fn1 \in (A\_27a^{ty\_2Enum\_2Enum}).(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ (ap\ V2fn1\ \mathsf{c\_2Enum\_2E0}))\ V0e))\ (ap\ (\mathsf{c\_2Ebool\_2E\_21}\ ty\_2Enum\_2Enum)\ (\lambda V3n \in ty\_2Enum\_2Enum.(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ (ap\ V2fn1\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V3n)))\ (ap\ (ap\ V1f\ (ap\ V2fn1\ V3n))\ V3n))))))))))$$