

thm_2EprimeFactor_2EDIVISOR__IN__PRIME__FACTORS
 (TMUEgvySFdygdczCBzEeCwUWVhRB-
 hUiFwKU)

October 26, 2020

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Enum_2ESUC_REP\ m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 9 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$.

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V0t)))$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A))$ of type $\iota \Rightarrow \iota$.

Definition 13 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.V0t)))$

Definition 14 We define $c_2Ebag_2EBAG_INSERT$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1b \in (ty_2Enum_2Enum^A_27a)$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 15 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40))))$

Definition 16 We define $c_2Edivides_2Edivides$ to be $\lambda V0a \in ty_2Enum_2Enum.\lambda V1b \in ty_2Enum_2Enum.V0a$

Definition 17 We define $c_2Emarker_2EAbbrev$ to be $\lambda V0x \in 2.V0x$.

Let $c_2EprimeFactor_2EPRIME_FACTORS : \iota$ be given. Assume the following.

$$c_2EprimeFactor_2EPRIME_FACTORS \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Ebag_2EITBAG : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Ebag_2EITBAG \\ A_27a A_27b \in (((A_27b^{A_27b})^{(ty_2Enum_2Enum^{A_27a})})^{(A_27b^{A_27b})^{A_27a}}) \end{aligned} \quad (9)$$

Definition 18 We define $c_2Ebag_2EBAG_GEN_PROD$ to be $\lambda V0bag \in (ty_2Enum_2Enum^{ty_2Enum_2Enum})$

Definition 19 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V0t)))$

Definition 20 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21))$

Definition 21 We define $c_Edivides_Eprime$ to be $\lambda V0a \in ty_2Enum_2Enum.(ap (ap c_Ebool_2E_2F_5C$

Definition 22 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 23 We define $c_Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 24 We define $c_Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 25 We define $c_Ebag_2EBAG_INN$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1n \in ty_2Enum_2Enum$

Definition 26 We define $c_Ebag_2EBAG_IN$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1b \in (ty_2Enum_2Enum$

Definition 27 We define $c_Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Definition 28 We define $c_Ebag_2EEMPTY_BAG$ to be $\lambda A_27a : \iota.(ap (c_Ecombin_2EK ty_2Enum_2Enum$

Definition 29 We define $c_Ebag_2EFINITE_BAG$ to be $\lambda A_27a : \iota.\lambda V0b \in (ty_2Enum_2Enum^{A_27a}).(ap$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & (ap (ap c_Earithmetic_2E_2A V0m) V1n) = (ap (ap c_Earithmetic_2E_2A \\ & V1n) V0m)))) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & (p (ap (ap c_Eprim_rec_2E_3C c_2Enum_2E0) (ap (ap c_Earithmetic_2E_2A \\ & V0m) V1n))) \Leftrightarrow ((p (ap (ap c_Eprim_rec_2E_3C c_2Enum_2E0) V0m)) \wedge \\ & (p (ap (ap c_Eprim_rec_2E_3C c_2Enum_2E0) V1n)))))) \end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0b \in (ty_2Enum_2Enum^{A_27a}). \\ & (\forall V1e1 \in A_27a.(\forall V2e2 \in A_27a.((p (ap (ap (c_Ebag_2EBAG_IN \\ & A_27a) V1e1) (ap (ap (c_Ebag_2EBAG_INSERT A_27a) V2e2) V0b))) \Leftrightarrow \\ & ((V1e1 = V2e2) \vee (p (ap (ap (c_Ebag_2EBAG_IN A_27a) V1e1) V0b)))))) \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0b \in (ty_2Enum_2Enum^{A_27a}). \\ & ((p (ap (c_Ebag_2EFINITE_BAG A_27a) V0b)) \Rightarrow (\forall V1e \in A_27a. \\ & (p (ap (c_Ebag_2EFINITE_BAG A_27a) (ap (ap (c_Ebag_2EBAG_INSERT \\ & A_27a) V1e) V0b)))))) \end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned} & (\forall V0b \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}).((p (ap (c_Ebag_2EFINITE_BAG \\ & ty_2Enum_2Enum) V0b)) \Rightarrow (\forall V1x \in ty_2Enum_2Enum.(\forall V2a \in \\ & ty_2Enum_2Enum.((ap (ap c_Ebag_2EBAG_GEN_PROD (ap (ap (c_Ebag_2EBAG_INSERT \\ & ty_2Enum_2Enum) V1x) V0b)) V2a) = (ap (ap c_Earithmetic_2E_2A \\ & V1x) (ap (ap c_Ebag_2EBAG_GEN_PROD V0b) V2a)))))) \end{aligned} \tag{14}$$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (V0x = V0x)) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in 2. (((\forall V2x \in A_27a. (p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in A_27a. ((p (ap V0P V3x)) \wedge (p V1Q)))))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). (((p V0P) \wedge (\forall V2x \in A_27a. (p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A_27a. ((p V0P) \wedge (p (ap V1Q V3x)))))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A_27a}). (((\forall V2x \in A_27a. ((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A_27a. (p (ap V1P V3x))) \vee (p V0Q)))))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \wedge (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C))))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (26)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0n)) \Rightarrow ((p (ap (c_2Ebag_2EFINITE_BAG ty_2Enum_2Enum) (ap c_2EprimeFactor_2EPRIME_FACTORS V0n))) \wedge ((\forall V1m \in ty_2Enum_2Enum. ((p (ap (ap (c_2Ebag_2EBAG_IN ty_2Enum_2Enum) V1m) (ap c_2EprimeFactor_2EPRIME_FACTORS V0n))) \Rightarrow (p (ap c_2Edivides_2Eprime V1m)))))) \wedge (V0n = (ap (ap c_2Ebag_2EBAG_GEN_PROD (ap c_2EprimeFactor_2EPRIME_FACTORS V0n)) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))) \quad (27)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0n)) \Rightarrow (\forall V1b \in (ty_2Enum_2Enum)^{ty_2Enum_2Enum}. (((p (ap (c_2Ebag_2EFINITE_BAG ty_2Enum_2Enum) V1b)) \wedge ((\forall V2x \in ty_2Enum_2Enum. ((p (ap (ap (c_2Ebag_2EBAG_IN ty_2Enum_2Enum) V2x) V1b)) \Rightarrow (p (ap c_2Edivides_2Eprime V2x)))))) \wedge ((ap (ap c_2Ebag_2EBAG_GEN_PROD V1b) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0n))) \Rightarrow (V1b = (ap c_2EprimeFactor_2EPRIME_FACTORS V0n)))))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (38)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (43)$$

Theorem 1

$$\begin{aligned} & (\forall V0p \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V1n)) \wedge ((p (ap c_2Edivides_2Eprime \\ & V0p)) \wedge (p (ap (ap c_2Edivides_2Edivides V0p) V1n)))) \Rightarrow (p (ap (ap \\ & (c_2Ebag_2EBAG_IN ty_2Enum_2Enum) V0p) (ap c_2EprimeFactor_2EPRIME_FACTORS \\ & V1n)))))) \end{aligned}$$