

thm\_2Eprobability\_2EPROB\_\_EQUIPROBABLE\_\_FINITE\_\_UNIO  
(TMW81ZamGmqbmTbD89eg1Y2V7jekKBWhYxp)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p$   
of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$   
of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

**Definition 4** We define  $c\_2Ebool\_2E\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V$

**Definition 5** We define  $c\_2Ecombin\_2E\_2EC$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 6** We define  $c\_2Ebool\_2E\_2E21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})$

**Definition 7** We define  $c\_2Ecombin\_2E\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Let  $c\_2Epred\_set\_2ECHOICE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Epred\_set\_2ECHOICE A\_27a \in (A\_27a^{(2^{A\_27a})}) \quad (1)$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty ty\_2Erealax\_2Ereal \quad (2)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (3)$$

Let  $c\_2Emeasure\_2Emeasure : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Emeasure A\_27a \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})(ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{(2^{A\_27a})}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})) \quad (4)$$

**Definition 8** We define  $c\_2Eprobability\_2Eprob$  to be  $\lambda A\_27a : \iota.(c\_2Emeasure\_2Emeasure A\_27a)$ .

**Definition 9** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 10** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 11** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

**Definition 12** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 13** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

**Definition 14** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A-27b})^{A-27a}}) \end{aligned} \quad (5)$$

**Definition 15** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2EABS\_prod A\_27a A\_27b) (V0x V1y))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a A\_27b \in ((2^{A-27a})^{((ty\_2Epair\_2Eprod A\_27a 2)^{A-27b})}) \end{aligned} \quad (6)$$

**Definition 16** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A-27a}).(ap (c\_2Epair\_2EABS\_prod A\_27a (2^{A-27a})) (V0x V1s))$

Let  $c\_2Emeasure\_2Emeasurable\_sets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_sets \\ A\_27a \in ((2^{(2^{A-27a})})^{(ty\_2Epair\_2Eprod (2^{A-27a}) (ty\_2Epair\_2Eprod (2^{(2^{A-27a})}) (ty\_2Erealax\_2Ereal^{(2^{A-27a})})))) \end{aligned} \quad (7)$$

**Definition 17** We define  $c\_2Eprobability\_2Eevents$  to be  $\lambda A\_27a : \iota.(c\_2Emeasure\_2Emeasurable\_sets A\_27a)$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (8)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (9)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (10)$$

**Definition 18** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 19** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (11)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (12)$$

**Definition 20** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (13)$$

**Definition 21** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 22** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (14)$$

Let  $c\_2Emeasure\_2Em\_space : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Em\_space\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))}) \quad (15)$$

**Definition 23** We define  $c\_2Eprobability\_2Ep\_space$  to be  $\lambda A\_27a : \iota.(c\_2Emeasure\_2Em\_space\ A\_27a)$ .

**Definition 24** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2ET)$ .

**Definition 25** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

**Definition 26** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_s$

Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (16)$$

**Definition 27** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2ET)$

**Definition 28** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 29** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 30** We define  $c\_2Earithmic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$ .

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (17)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal\_REP\_CLASS}) \quad (18)$$

**Definition 31** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ (t$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (19)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (20)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (21)$$

**Definition 32** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 33** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal})^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (22)$$

**Definition 34** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 35** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (23)$$

**Definition 36** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 37** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 38** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

**Definition 39** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealx\_2Ereal.(ap (ap (ap (c\_2Ebool\_2ECONJ$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \end{aligned} \quad (24)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \end{aligned} \quad (25)$$

**Definition 40** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a}$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Emetric\_2Emetric A0) \quad (26)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emetric\_2Emetric A\_27a \in ((ty\_2Emetric\_2Emetric \\ A\_27a)^{(ty\_2Erealx\_2Ereal^{(ty\_2Epair\_2Eprod A\_27a A\_27a)})}) \end{aligned} \quad (27)$$

**Definition 41** We define  $c\_2Emetric\_2Emr1$  to be  $(ap (c\_2Emetric\_2Emetric ty\_2Erealx\_2Ereal) (ap (c$

Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emetric\_2Edist A\_27a \in ((ty\_2Erealx\_2Ereal^{(ty\_2Epair\_2Eprod A\_27a A\_27a)}) \quad (28)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Etopology\_2Etopology A0) \quad (29)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Etopology\_2Etopology A\_27a \in \\ ((ty\_2Etopology\_2Etopology A\_27a)^{(2^{(2^{A\_27a})})}) \end{aligned} \quad (30)$$

**Definition 42** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric A\_27a).(ap$

Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Enets\_2Etends \\ A\_27a A\_27b \in (((2^{(ty\_2Epair\_2Eprod (ty\_2Etopology\_2Etopology A\_27a) ((2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a^{A\_27b})}) \end{aligned} \quad (31)$$

**Definition 43** We define  $c\_2Eseq\_2E\_2D\_2D\_3E$  to be  $\lambda V0x \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

**Definition 44** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_2$

**Definition 45** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2$

**Definition 46** We define  $c\_2Epred\_set\_2EDISJOINT$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2$

**Definition 47** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in (2^{A\_27b}).(ap (c\_2$

**Definition 48** We define  $c\_2Emeasure\_2Ecountably\_additive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{2^{A\_27a}}))$

**Definition 49** We define  $c\_2Emeasure\_2Epositive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{2^{A\_27a}}))$

Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Esubsets A\_27a \in ( (2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{2^{A\_27a}}))}) \quad (32)$$

**Definition 50** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2$

**Definition 51** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A\_27a}).(ap (c\_2$

**Definition 52** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E3F$

**Definition 53** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2$

Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Espace A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{2^{A\_27a}}))}) \quad (33)$$

**Definition 54** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2$

**Definition 55** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27a})})$

**Definition 56** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{2^{A\_27a}}))$

**Definition 57** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{2^{A\_27a}}))$

**Definition 58** We define  $c\_2Emeasure\_2Emeasure\_space$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{2^{A\_27a}}))$

**Definition 59** We define  $c\_2Eprobability\_2Eprob\_space$  to be  $\lambda A\_27a : \iota.\lambda V0p \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{2^{A\_27a}}))$

Let  $c\_2Epred\_set\_2ECARD : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Epred\_set\_2ECARD A\_27a \in (ty\_2Enum\_2Enum^{(2^{A\_27a})}) \quad (34)$$

Let  $c\_2Erealx\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etrealmul \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal (ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal})^{ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal})^{ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal}) \quad (35)$$

**Definition 60** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Epred\_set\_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EITSET \\ A\_27a\ A\_27b \in (((A\_27b^{A\_27a})^{(2^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})}) \end{aligned} \quad (36)$$

**Definition 61** We define  $c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal$

**Definition 62** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E\_21\ (2$

Assume the following.

$$True \quad (37)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (40)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (41)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} ((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (44)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (45)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t))))) \quad (47)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \wedge (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (49)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3))))) \quad (50)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27))))))) \quad (51)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). ((\neg(V0s = (c.2Epred\_set\_2EEMPTY\ A\_27a))) \Rightarrow (p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A\_27a)\ (ap\ (c.2Epred\_set\_2ECHOICE\ A\_27a)\ V0s))\ V0s)))) \quad (52)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((ap\ (c.2Epred\_set\_2ECARD\ A\_27a)\ (c.2Epred\_set\_2EEMPTY\ A\_27a)) = c.2Enum\_2E0) \quad (53)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))). ((p\ (ap\ (c.2Eprobability\_2Eprob\_space\ A\_27a)\ V0p)) \Rightarrow ((ap\ (ap\ (c.2Eprobability\_2Eprob\ A\_27a)\ V0p)\ (c.2Epred\_set\_2EEMPTY\ A\_27a)) = (ap\ c.2Ereal\_2Ereal\_of\_num\ c.2Enum\_2E0)))) \quad (54)$$



Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod \\
& (2^{A.27a}) (ty\_2Epair\_2Eprod (2^{(2^{A.27a})}) (ty\_2Erealax\_2Ereal^{(2^{A.27a})}))))). \\
& (\forall V1s \in (2^{A.27a}).(((p (ap (c\_2Eprobability\_2Eprob\_space \\
A.27a) V0p)) \wedge ((p (ap (ap (c\_2Ebool\_2EIN (2^{A.27a}) V1s) (ap (c\_2Eprobability\_2Eevents \\
A.27a) V0p)))) \wedge ((\forall V2x \in A.27a.((p (ap (ap (c\_2Ebool\_2EIN \\
A.27a) V2x) V1s)) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN (2^{A.27a}) (ap (ap ( \\
c\_2Epred\_set\_2EINSERT A.27a) V2x) (c\_2Epred\_set\_2EEMPTY A.27a))) \\
(ap (c\_2Eprobability\_2Eevents A.27a) V0p)))))) \wedge (p (ap (c\_2Epred\_set\_2EFINITE \\
A.27a) V1s)))))) \Rightarrow ((ap (ap (c\_2Eprobability\_2Eprob A.27a) V0p) \\
V1s) = (ap (ap (c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE A.27a) (\lambda V3x \in \\
A.27a.(ap (ap (c\_2Eprobability\_2Eprob A.27a) V0p) (ap (ap (c\_2Epred\_set\_2EINSERT \\
A.27a) V3x) (c\_2Epred\_set\_2EEMPTY A.27a)))))) V1s))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_mul \\
(ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num \\
c\_2Enum\_2E0)))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.( \\
& (((ap c\_2Ereal\_2Ereal\_of\_num V0n) = (ap c\_2Ereal\_2Ereal\_of\_num \\
V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap c\_2Erealax\_2Ereal\_neg (ap c\_2Ereal\_2Ereal\_of\_num \\
V0n)) = (ap c\_2Ereal\_2Ereal\_of\_num V1m)) \Leftrightarrow ((V0n = c\_2Enum\_2E0) \wedge \\
(V1m = c\_2Enum\_2E0))) \wedge (((ap c\_2Ereal\_2Ereal\_of\_num V0n) = \\
(ap c\_2Erealax\_2Ereal\_neg (ap c\_2Ereal\_2Ereal\_of\_num V1m))) \Leftrightarrow \\
((V0n = c\_2Enum\_2E0) \wedge (V1m = c\_2Enum\_2E0))) \wedge (((ap c\_2Erealax\_2Ereal\_neg \\
(ap c\_2Ereal\_2Ereal\_of\_num V0n)) = (ap c\_2Erealax\_2Ereal\_neg \\
(ap c\_2Ereal\_2Ereal\_of\_num V1m))) \Leftrightarrow (V0n = V1m))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((p (ap \\
(c\_2Epred\_set\_2EFINITE A.27a) V0P)) \Rightarrow (\forall V1f \in (ty\_2Erealax\_2Ereal^{A.27a}). \\
(\forall V2p \in A.27a.(((p (ap (ap (c\_2Ebool\_2EIN A.27a) V2p) V0P)) \wedge \\
(\forall V3q \in A.27a.((p (ap (ap (c\_2Ebool\_2EIN A.27a) V3q) V0P)) \Rightarrow \\
((ap V1f V2p) = (ap V1f V3q)))))) \Rightarrow ((ap (ap (c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE \\
A.27a) V1f) V0P) = (ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Ereal\_2Ereal\_of\_num \\
(ap (c\_2Epred\_set\_2ECARD A.27a) V0P))) (ap V1f V2p)))))))))
\end{aligned} \tag{58}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{59}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{60}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (61)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (62)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (69)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (70)$$

**Theorem 1**

$$\begin{aligned}
& \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod \\
& (2^{A_{.27a}}) (ty\_2Epair\_2Eprod (2^{(2^{A_{.27a}})}) (ty\_2Erealax\_2Ereal(2^{A_{.27a}}))))). \\
& (\forall V1s \in (2^{A_{.27a}}). (((p (ap (c\_2Eprobability\_2Eprob\_space \\
& A_{.27a}) V0p)) \wedge ((p (ap (ap (c\_2Ebool\_2EIN (2^{A_{.27a}}) V1s) (ap (c\_2Eprobability\_2Eevents \\
& A_{.27a}) V0p))) \wedge ((\forall V2x \in A_{.27a}. ((p (ap (ap (c\_2Ebool\_2EIN \\
& A_{.27a}) V2x) V1s)) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN (2^{A_{.27a}}) (ap (ap ( \\
& c\_2Epred\_set\_2EINSERT A_{.27a}) V2x) (c\_2Epred\_set\_2EEMPTY A_{.27a}))) \\
& (ap (c\_2Eprobability\_2Eevents A_{.27a}) V0p)))))) \wedge ((p (ap (c\_2Epred\_set\_2EFINITE \\
& A_{.27a}) V1s)) \wedge (\forall V3x \in A_{.27a}. (\forall V4y \in A_{.27a}. (((p (ap \\
& (ap (c\_2Ebool\_2EIN A_{.27a}) V3x) V1s)) \wedge (p (ap (ap (c\_2Ebool\_2EIN \\
& A_{.27a}) V4y) V1s)))) \Rightarrow ((ap (ap (c\_2Eprobability\_2Eprob A_{.27a}) V0p) \\
& (ap (ap (c\_2Epred\_set\_2EINSERT A_{.27a}) V3x) (c\_2Epred\_set\_2EEMPTY \\
& A_{.27a}))) = (ap (ap (c\_2Eprobability\_2Eprob A_{.27a}) V0p) (ap (ap ( \\
& c\_2Epred\_set\_2EINSERT A_{.27a}) V4y) (c\_2Epred\_set\_2EEMPTY A_{.27a})))))))))) \Rightarrow \\
& ((ap (ap (c\_2Eprobability\_2Eprob A_{.27a}) V0p) V1s) = (ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap (c\_2Epred\_set\_2ECARD A_{.27a}) \\
& V1s))) (ap (ap (c\_2Eprobability\_2Eprob A_{.27a}) V0p) (ap (ap (c\_2Epred\_set\_2EINSERT \\
& A_{.27a}) (ap (c\_2Epred\_set\_2ECHOICE A_{.27a}) V1s)) (c\_2Epred\_set\_2EEMPTY \\
& A_{.27a})))))))))
\end{aligned}$$