

thm\_2Eprobability\_2EPROB\_\_REAL\_\_SUM\_\_IMAGE\_\_FN  
 (TMdfcdhfaqKSoVGKoozaB-  
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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap P x))$  **then** (the  $(\lambda x.x \in A \wedge p$   
of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$   
of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2EBOUNDED$  to be  $(\lambda V0v \in 2.c\_2Ebool\_2E\_ET)$ .

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})$

**Definition 6** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A-27c}).\lambda V1g$

**Definition 7** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

**Definition 8** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow q Q)$   
of type  $\iota$ .

**Definition 9** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c$

**Definition 10** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A-27b})^{A-27a}}) \tag{2}$$

**Definition 11** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \quad (3)$$

**Definition 12** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Emin\_2E\_2F$

**Definition 13** We define  $c\_2Ebool\_2E\_2F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap (c\_2Emin\_2E\_2F$

**Definition 14** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

**Definition 15** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap (c\_2Epred\_set\_2E$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (4)$$

Let  $c\_2Emeasure\_2Em\_space : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Em\_space\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))) \quad (5)$$

Let  $c\_2Emeasure\_2Emeasurable\_sets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_sets\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))) \quad (6)$$

Let  $c\_2Emeasure\_2Emeasure : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasure\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))) \quad (7)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (8)$$

**Definition 16** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E\_2ET)$ .

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (9)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (10)$$

**Definition 17** We define  $c\_Enum\_E0$  to be  $(ap\ c\_Enum\_EABS\_num\ c\_Enum\_EZERO\_REP)$ .

Let  $c\_Ereal\_Esum : \iota$  be given. Assume the following.

$$c\_Ereal\_Esum \in ((ty\_Erealax\_Ereal^{(ty\_Erealax\_Ereal^{ty\_Eenum\_Eenum})})^{(ty\_Epair\_Eprod\ ty\_Eenum\_Eenum)}) \quad (11)$$

**Definition 18** We define  $c\_Ebool\_EF$  to be  $(ap\ (c\_Ebool\_E21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 19** We define  $c\_Ebool\_E7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_Emin\_E3D\_3D\_3E\ V0t)\ c\_Ebool\_E21))$

Let  $c\_Enum\_EREP\_num : \iota$  be given. Assume the following.

$$c\_Enum\_EREP\_num \in (\omega^{ty\_Eenum\_Eenum}) \quad (12)$$

Let  $c\_Enum\_ESUC\_REP : \iota$  be given. Assume the following.

$$c\_Enum\_ESUC\_REP \in (\omega^{\omega}) \quad (13)$$

**Definition 20** We define  $c\_Enum\_ESUC$  to be  $\lambda V0m \in ty\_Eenum\_Eenum.(ap\ c\_Enum\_EABS\_num\ m)$

**Definition 21** We define  $c\_Eprim\_rec\_E3C$  to be  $\lambda V0m \in ty\_Eenum\_Eenum.\lambda V1n \in ty\_Eenum\_Eenum$

**Definition 22** We define  $c\_Earithmic\_E3E$  to be  $\lambda V0m \in ty\_Eenum\_Eenum.\lambda V1n \in ty\_Eenum\_Eenum$

**Definition 23** We define  $c\_Ebool\_E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_Ebool\_E21\ 2)\ (\lambda V2t \in 2.V2t))))$

**Definition 24** We define  $c\_Earithmic\_E3E\_3D$  to be  $\lambda V0m \in ty\_Eenum\_Eenum.\lambda V1n \in ty\_Eenum\_Eenum$

Let  $ty\_Ehreal\_Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_Ehreal\_Ehreal \quad (14)$$

Let  $c\_Erealax\_Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_REP\_CLASS \in ((2^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})^{(ty\_Erealax\_Ereal)}) \quad (15)$$

**Definition 25** We define  $c\_Erealax\_Ereal\_REP$  to be  $\lambda V0a \in ty\_Erealax\_Ereal.(ap\ (c\_Emin\_E40\ t)\ a)$

Let  $c\_Erealax\_Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_neg \in ((ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)}) \quad (16)$$

Let  $c\_Erealax\_Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_eq \in ((2^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)}) \quad (17)$$

Let  $c\_Erealax\_Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_ABS\_CLASS \in (ty\_Erealax\_Ereal^{(2^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})}) \quad (18)$$

**Definition 26** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 27** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal)$

Let  $c\_2Erealax\_2Ereal\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)} \quad (19)$$

**Definition 28** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 29** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}) \quad (20)$$

Let  $c\_2Erealax\_2Ereal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)} \quad (21)$$

**Definition 30** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 31** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 32** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 33** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (22)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (23)$$

**Definition 34** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \quad (24)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \quad (25)$$

**Definition 35** We define  $c\_2Emetric\_2Emr1$  to be  $(ap (c\_2Emetric\_2Emetric ty\_2Erealx\_2Ereal) (ap (c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emetric\_2Edist A\_27a \in ((ty\_2Erealx\_2Ereal^{(ty\_2Epair\_2Eprod A\_27a A\_27b)})_{A\_27a})_{A\_27a} \quad (26)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Etopology\_2Etopology A0) \quad (27)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Etopology\_2Etopology A\_27a \in ((ty\_2Etopology\_2Etopology A\_27a)^{(2^{(2^A-27a)})})_{A\_27a} \quad (28)$$

**Definition 36** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Emetric\_2Emetric A\_27a). (ap (c\_2Emetric\_2Edist : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Enets\_2Etends A\_27a A\_27b \in (((2^{(ty\_2Epair\_2Eprod (ty\_2Etopology\_2Etopology A\_27a) ((2^{A-27b})^{A-27b}))})_{A\_27a})_{(A\_27a)^{A-27b}})_{A\_27a} \quad (29)$$

**Definition 37** We define  $c\_2Eseq\_2E\_2D\_2D\_2E$  to be  $\lambda V0x \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}). \lambda V1x \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}). \lambda V1s \in ty\_2Enum$

**Definition 38** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}). \lambda V1s \in ty\_2Enum$

**Definition 39** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 40** We define  $c\_2Epred\_set\_2EDISJOINT$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c\_2Ebool\_2EF))$

**Definition 41** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0P \in (2^{A-27a}). \lambda V1Q \in (2^{A-27b}). (ap (c\_2Ebool\_2EF))$

**Definition 42** We define  $c\_2Emeasure\_2Ecountably\_additive$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Epair\_2Eprod (2^{A-27a}) (2^{2^{A-27a}}))$

**Definition 43** We define  $c\_2Emeasure\_2Epositive$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Epair\_2Eprod (2^{A-27a}) (2^{2^{A-27a}}))$

Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Esubsets A\_27a \in ((2^{(2^{A-27a})})^{(ty\_2Epair\_2Eprod (2^{A-27a}) (2^{2^{A-27a}}))})_{A\_27a} \quad (30)$$

**Definition 44** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b)^{A-27a}. \lambda V1s \in (A\_27a)^{A-27b}$

**Definition 45** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). (ap (c\_2Ebool\_2E3F))$

**Definition 46** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c\_2Ebool\_2EF))$

Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Espace\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})})})) \quad (31)$$

**Definition 47** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2E$

**Definition 48** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27a})})$

**Definition 49** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 50** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 51** We define  $c\_2Emeasure\_2Emeasure\_space$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 52** We define  $c\_2Eprobability\_2Eprob\_space$  to be  $\lambda A\_27a : \iota.(c\_2Emeasure\_2Em\_space\ A\_27a)$ .

**Definition 53** We define  $c\_2Eprobability\_2Eprob$  to be  $\lambda A\_27a : \iota.(c\_2Emeasure\_2Emeasure\ A\_27a)$ .

**Definition 54** We define  $c\_2Eprobability\_2Eevents$  to be  $\lambda A\_27a : \iota.(c\_2Emeasure\_2Emeasurable\_sets\ A\_27a)$

**Definition 55** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (32)$$

**Definition 56** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ n))$

**Definition 57** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 58** We define  $c\_2Eprobability\_2Eprob\_space$  to be  $\lambda A\_27a : \iota.\lambda V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 59** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_2E$

**Definition 60** We define  $c\_2Epred\_set\_2EDELETE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap\ (ap\ (c\_2E$

Let  $c\_2Epred\_set\_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EITSET\ A\_27a\ A\_27b \in (((A\_27b^{A\_27b})^{(2^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})}) \quad (33)$$

**Definition 61** We define  $c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Ereal\_x\_2E$

**Definition 62** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E21\ (2^{A\_27a}))$

Assume the following.

$$True \quad (34)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (37)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (41)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (42)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (V0x = V0x)) \quad (43)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (44)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (45)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27b^{A\_27a}). ((V0f = V1g) \Leftrightarrow (\forall V2x \in A\_27a. ((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (47)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (48)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). ((\neg(\exists V1x \in A\_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (49)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). (((p\ V0P) \wedge (\forall V2x \in A\_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \quad (50)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A\_27a}). ((\forall V2x \in A\_27a. ((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a. (p\ (ap\ V1P\ V3x)) \vee (p\ V0Q)))))) \quad (51)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). ((\forall V2x \in A\_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \quad (53)$$



Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \wedge (p V2C)))))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C))) \quad (54)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)))))) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A))) \quad (55)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3)))))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))) \quad (56)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow ( \\ & \quad \forall V0b \in 2.(\forall V1f \in (A\_27b^{A\_27a}).(\forall V2g \in (A\_27b^{A\_27a}). \\ & \quad (\forall V3x \in A\_27a.((ap (ap (ap (ap (c\_2Ebool\_2ECOND (A\_27b^{A\_27a}) \\ V0b) V1f) V2g) V3x) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27b) V0b) (ap \\ V1f V3x)) (ap V2g V3x))))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow ( \\ & \quad \forall V0f \in (A\_27b^{A\_27a}).(\forall V1b \in 2.(\forall V2x \in A\_27a. \\ & \quad (\forall V3y \in A\_27a.((ap V0f (ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) \\ V1b) V2x) V3y) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27b) V1b) (ap V0f \\ V2x)) (ap V0f V3y))))))) \end{aligned} \quad (58)$$

Assume the following.

$$2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow 2.(((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))) \quad (59)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & \quad (\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a. \\ & \quad (\forall V5y\_27 \in A\_27a.(((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & \quad ((\neg (p V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) \\ V0P) V2x) V4y) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) V1Q) V3x\_27) \\ V5y\_27)))))) \end{aligned} \quad (60)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p (ap V0P V2x)))))) \Leftrightarrow (p (ap V0P V1a)))) \quad (61)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & ((\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a. (\forall V3t2 \in A\_27a. ((ap \\ & (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V2t1)\ V3t2) = V3t2)))))) \end{aligned} \quad (62)$$

Assume the following.

$$(\forall V0v \in 2. ((p\ (ap\ c\_2Ebool\_2EBOUNDED\ V0v)) \Leftrightarrow True)) \quad (63)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & (\forall V0m \in (ty\_2Epair\_2Eprod \\ & (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))))). \\ & (\forall V1s \in (2^{A\_27a}). (((p\ (ap\ (c\_2Emeasure\_2Emeasure\_space \\ & A\_27a)\ V0m)) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A\_27a})\ V1s)\ (ap\ (c\_2Emeasure\_2Emeasurable\_sets \\ & A\_27a)\ V0m)))))) \Rightarrow (p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ A\_27a)\ V1s) \\ & (ap\ (c\_2Emeasure\_2Em\_space\ A\_27a)\ V0m)))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\ & (2^{A\_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V1t))))))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & (\forall V0x \in A\_27a. (\neg (p\ (ap\ (ap \\ & (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (c\_2Epred\_set\_2EEMPTY\ A\_27a)))))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\ & (2^{A\_27a}). (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\ & V2x)\ (ap\ (ap\ (c\_2Epred\_set\_2EUNION\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\ & (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & A\_27a)\ V2x)\ V1t))))))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\ & (2^{A\_27a}). (\forall V2u \in (2^{A\_27a}). ((ap\ (ap\ (c\_2Epred\_set\_2EUNION \\ & A\_27a)\ V0s)\ (ap\ (ap\ (c\_2Epred\_set\_2EUNION\ A\_27a)\ V1t)\ V2u)) = ( \\ & ap\ (ap\ (c\_2Epred\_set\_2EUNION\ A\_27a)\ (ap\ (ap\ (c\_2Epred\_set\_2EUNION \\ & A\_27a)\ V0s)\ V1t))\ V2u)))))) \end{aligned} \quad (68)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A-27a}).(\forall V1t \in (2^{A-27a}).((ap\ (ap\ (c.2Epred\_set\_2EUNION\ A.27a\ V0s)\ V1t) = (ap\ (ap\ (c.2Epred\_set\_2EUNION\ A.27a\ V1t)\ V0s)))))) \quad (69)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A-27a}).(\forall V1t \in (2^{A-27a}).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a\ V2x)\ (ap\ (ap\ (c.2Epred\_set\_2EINTER\ A.27a\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a\ V2x)\ V1t)))))))))) \quad (70)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0s \in (2^{A-27a}).((ap\ (ap\ (c.2Epred\_set\_2EINTER\ A.27a)\ (c.2Epred\_set\_2EEMPTY\ A.27a)\ V0s) = (c.2Epred\_set\_2EEMPTY\ A.27a))) \wedge (\forall V1s \in (2^{A-27a}).((ap\ (ap\ (c.2Epred\_set\_2EINTER\ A.27a)\ V1s)\ (c.2Epred\_set\_2EEMPTY\ A.27a)) = (c.2Epred\_set\_2EEMPTY\ A.27a)))))) \quad (71)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A-27a}).(\forall V1t \in (2^{A-27a}).((p\ (ap\ (ap\ (c.2Epred\_set\_2EDISJOINT\ A.27a\ V0s)\ V1t)) \Leftrightarrow (\neg(\exists V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a\ V2x)\ V1t)))))))))) \quad (72)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A-27a}).(\forall V1t \in (2^{A-27a}).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a\ V2x)\ (ap\ (ap\ (c.2Epred\_set\_2EDIFF\ A.27a\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a\ V2x)\ V0s)) \wedge (\neg(p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a\ V2x)\ V1t)))))))))) \quad (73)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.(\forall V2s \in (2^{A-27a}).((p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a\ V0x)\ (ap\ (ap\ (c.2Epred\_set\_2EINSERT\ A.27a\ V1y)\ V2s))) \Leftrightarrow ((V0x = V1y) \vee (p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a\ V0x)\ V2s)))))))))) \quad (74)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A-27a}).((V0s = (c.2Epred\_set\_2EEMPTY\ A.27a)) \vee (\exists V1x \in A.27a.(\exists V2t \in (2^{A-27a}).((V0s = (ap\ (ap\ (c.2Epred\_set\_2EINSERT\ A.27a\ V1x)\ V2t)) \wedge (\neg(p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a\ V1x)\ V2t)))))))))) \quad (75)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1s \in \\ (2^{A\_27a}). ((\neg(p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ V1s))) \Leftrightarrow ((ap \\ (ap\ (c\_2Epred\_set\_2EDELETE\ A\_27a)\ V1s)\ V0x) = V1s)))) \end{aligned} \quad (76)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\ A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT \\ A\_27a)\ V1y)\ (c\_2Epred\_set\_2EEMPTY\ A\_27a)))) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0y \in A\_27b. (\forall V1s \in (2^{A\_27a}). (\forall V2f \in (A\_27b^{A\_27a}). \\ ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27b)\ V0y)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\ A\_27a\ A\_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A\_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V1s)))))) \end{aligned} \quad (78)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0f \in (A\_27b^{A\_27a}). ((ap\ (ap\ (c\_2Epred\_set\_2EIMAGE\ A\_27a \\ A\_27b)\ V0f)\ (c\_2Epred\_set\_2EEMPTY\ A\_27a)) = (c\_2Epred\_set\_2EEMPTY \\ A\_27b))) \end{aligned} \quad (79)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0f \in (A\_27b^{A\_27a}). (\forall V1x \in A\_27a. (\forall V2s \in ( \\ 2^{A\_27a}). ((ap\ (ap\ (c\_2Epred\_set\_2EIMAGE\ A\_27a\ A\_27b)\ V0f)\ (ap \\ (ap\ (c\_2Epred\_set\_2EINSERT\ A\_27a)\ V1x)\ V2s)) = (ap\ (ap\ (c\_2Epred\_set\_2EINSERT \\ A\_27b)\ (ap\ V0f\ V1x))\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE\ A\_27a\ A\_27b) \\ V0f)\ V2s)))))) \end{aligned} \quad (80)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(2^{A\_27a})}). (( \\ (p\ (ap\ V0P\ (c\_2Epred\_set\_2EEMPTY\ A\_27a))) \wedge (\forall V1s \in (2^{A\_27a}). \\ (((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ V1s)) \wedge (p\ (ap\ V0P\ V1s))) \Rightarrow \\ (\forall V2e \in A\_27a. ((\neg(p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2e)\ V1s))) \Rightarrow \\ (p\ (ap\ V0P\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ A\_27a)\ V2e)\ V1s)))))) \Rightarrow \\ (\forall V3s \in (2^{A\_27a}). ((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a) \\ V3s)) \Rightarrow (p\ (ap\ V0P\ V3s)))))) \end{aligned} \quad (81)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1s \in \\ & (2^{A.27a}). ((p\ (ap\ (c.2Epred\_set\_2EFINITE\ A.27a)\ (ap\ (ap\ (c.2Epred\_set\_2EINSERT \\ & A.27a)\ V0x)\ V1s))) \Leftrightarrow (p\ (ap\ (c.2Epred\_set\_2EFINITE\ A.27a)\ V1s)))) \end{aligned} \quad (82)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow ((ap\ (c.2Epred\_set\_2EBIGUNION \\ & A.27a)\ (c.2Epred\_set\_2EEMPTY\ (2^{A.27a}))) = (c.2Epred\_set\_2EEMPTY \\ & A.27a)) \end{aligned} \quad (83)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in (2^{A.27a}). ((ap\ (c.2Epred\_set\_2EBIGUNION \\ & A.27a)\ (ap\ (ap\ (c.2Epred\_set\_2EINSERT\ (2^{A.27a})\ V0x)\ (c.2Epred\_set\_2EEMPTY \\ & (2^{A.27a})))) = V0x)) \end{aligned} \quad (84)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1P \in \\ & (2^{(2^{A.27a})}). ((ap\ (c.2Epred\_set\_2EBIGUNION\ A.27a)\ (ap\ (ap \\ & (c.2Epred\_set\_2EINSERT\ (2^{A.27a})\ V0s)\ V1P))) = (ap\ (ap\ (c.2Epred\_set\_2EUNION \\ & A.27a)\ V0s)\ (ap\ (c.2Epred\_set\_2EBIGUNION\ A.27a)\ V1P)))))) \end{aligned} \quad (85)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod \\ & (2^{A.27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A.27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A.27a})}))). \\ & ((p\ (ap\ (c.2Eprobability\_2Eprob\_space\ A.27a)\ V0p)) \Rightarrow ((ap\ (ap \\ & (c.2Eprobability\_2Eprob\ A.27a)\ V0p)\ (c.2Epred\_set\_2EEMPTY \\ & A.27a)) = (ap\ c.2Ereal\_2Ereal\_of\_num\ c.2Enum\_2E0)))) \end{aligned} \quad (86)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod \\ & (2^{A.27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A.27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A.27a})}))). \\ & ((p\ (ap\ (c.2Eprobability\_2Eprob\_space\ A.27a)\ V0p)) \Rightarrow ((ap\ (ap \\ & (c.2Eprobability\_2Eprob\ A.27a)\ V0p)\ (ap\ (c.2Eprobability\_2Eprob\_space \\ & A.27a)\ V0p)) = (ap\ c.2Ereal\_2Ereal\_of\_num\ (ap\ c.2Earithmetic\_2ENUMERAL \\ & (ap\ c.2Earithmetic\_2EBIT1\ c.2Earithmetic\_2EZERO)))))) \end{aligned} \quad (87)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod \\
& (2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{(2^{A\_27a})}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))))). \\
& (\forall V1s \in (2^{A\_27a}).(\forall V2t \in (2^{A\_27a}).(\forall V3u \in \\
& (2^{A\_27a}).(((p (ap (c\_2Eprobability\_2Eprob\_space\ A\_27a)\ V0p)) \wedge \\
& ((p (ap (ap (c\_2Ebool\_2EIN (2^{A\_27a})\ V1s) (ap (c\_2Eprobability\_2Eevents \\
& A\_27a)\ V0p))) \wedge ((p (ap (ap (c\_2Ebool\_2EIN (2^{A\_27a})\ V2t) (ap (c\_2Eprobability\_2Eevents \\
& A\_27a)\ V0p))) \wedge ((p (ap (ap (c\_2Epred\_set\_2EDISJOINT\ A\_27a)\ V1s) \\
& V2t)) \wedge (V3u = (ap (ap (c\_2Epred\_set\_2EUNION\ A\_27a)\ V1s)\ V2t)))))) \Rightarrow \\
& ((ap (ap (c\_2Eprobability\_2Eprob\ A\_27a)\ V0p)\ V3u) = (ap (ap (c\_2Erealax\_2Ereal\_add \\
& (ap (ap (c\_2Eprobability\_2Eprob\ A\_27a)\ V0p)\ V1s) (ap (ap (c\_2Eprobability\_2Eprob \\
& A\_27a)\ V0p)\ V2t)))))))))
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod \\
& (2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{(2^{A\_27a})}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))))). \\
& (\forall V1s \in (2^{A\_27a}).(\forall V2t \in (2^{A\_27a}).(((p (ap (c\_2Eprobability\_2Eprob\_space \\
& A\_27a)\ V0p)) \wedge ((p (ap (ap (c\_2Ebool\_2EIN (2^{A\_27a})\ V1s) (ap (c\_2Eprobability\_2Eevents \\
& A\_27a)\ V0p))) \wedge (p (ap (ap (c\_2Ebool\_2EIN (2^{A\_27a})\ V2t) (ap (c\_2Eprobability\_2Eevents \\
& A\_27a)\ V0p)))))) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN (2^{A\_27a}) (ap (ap (c\_2Epred\_set\_2EDIFF \\
& A\_27a)\ V1s)\ V2t) (ap (c\_2Eprobability\_2Eevents\ A\_27a)\ V0p)))))))))
\end{aligned} \tag{89}$$

Assume the following.

$$\begin{aligned}
& (\neg((ap\ c\_2Ereal\_2Ereal\_of\_num (ap\ c\_2Earithmetic\_2ENUMERAL \\
& (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))) = (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)))
\end{aligned} \tag{90}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal.(((ap (ap\ c\_2Erealax\_2Ereal\_add \\
& V0x)\ V1y) = (ap (ap\ c\_2Erealax\_2Ereal\_add\ V0x)\ V2z)) \Leftrightarrow (V1y = V2z))))))
\end{aligned} \tag{91}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Erealax\_2Ereal^{A\_27a}). \\
& (((ap (ap (c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE\ A\_27a)\ V0f) (c\_2Epred\_set\_2EEMPTY \\
& A\_27a)) = (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)) \wedge (\forall V1e \in \\
& A\_27a.(\forall V2s \in (2^{A\_27a}).((p (ap (c\_2Epred\_set\_2EFINITE \\
& A\_27a)\ V2s)) \Rightarrow ((ap (ap (c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE\ A\_27a) \\
& V0f) (ap (ap (c\_2Epred\_set\_2EINSERT\ A\_27a)\ V1e)\ V2s)) = (ap (ap \\
& c\_2Erealax\_2Ereal\_add (ap\ V0f\ V1e)) (ap (ap (c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE \\
& A\_27a)\ V0f) (ap (ap (c\_2Epred\_set\_2EDELETE\ A\_27a)\ V2s)\ V1e)))))))))
\end{aligned} \tag{92}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((p (ap \\
& (c.2Epred\_set\_2EFINITE A.27a) V0P)) \Rightarrow (\forall V1f \in (ty.2Erealax.2Ereal^{A.27a}). \\
& ((ap (ap (c.2Ereal\_sigma\_2EREAL\_SUM\_IMAGE A.27a) V1f) V0P) = \\
& (ap (ap (c.2Ereal\_sigma\_2EREAL\_SUM\_IMAGE A.27a) (\lambda V2x \in \\
& A.27a.(ap (ap (ap (c.2Ebool.2ECOND ty.2Erealax.2Ereal) (ap (ap \\
& (c.2Ebool.2EIN A.27a) V2x) V0P)) (ap V1f V2x)) (ap c.2Ereal.2Ereal\_of\_num \\
& c.2Enum.2E0)))))) V0P))))))
\end{aligned} \tag{93}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{94}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{95}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{96}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{97}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{98}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{100}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r)))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{101}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge ( \\
& \neg(p V1q)) \vee ((p V2r) \vee \neg(p V0p))))))))) \\
& \tag{102}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge (\neg(p V1q)) \vee \neg(p V0p)))))) \\
& \tag{103}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (\forall V3s \in \\
& 2. (((p V0p) \Leftrightarrow (p (ap (ap (ap (c_2Ebool_2ECOND 2) V1q) V2r) V3s))) \Leftrightarrow \\
& (((p V0p) \vee ((p V1q) \vee \neg(p V3s)))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee \neg(p V1q))) \wedge \\
& (((p V0p) \vee (\neg(p V2r)) \vee \neg(p V3s)))) \wedge (((\neg(p V1q)) \vee ((p V2r) \vee \neg( \\
& p V0p)))) \wedge ((p V1q) \vee ((p V3s) \vee \neg(p V0p))))))))) \\
& \tag{104}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \\
& \tag{105}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q))) \\
& \tag{106}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p))) \\
& \tag{107}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q))) \\
& \tag{108}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\neg(\neg(p V0p))) \Rightarrow (p V0p)) \\
& \tag{109}
\end{aligned}$$



**Theorem 1**

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow ( \\
& \forall V0p \in (ty\_2Epair\_2Eprod\ (2^{A_{.27a}})\ (ty\_2Epair\_2Eprod\ ( \\
& \quad 2^{(2^{A_{.27a}})})\ (ty\_2Erealax\_2Ereal^{(2^{A_{.27a}})}))) . (\forall V1f \in \\
& \quad ((2^{A_{.27a}})^{A_{.27b}}) . (\forall V2e \in (2^{A_{.27a}}) . (\forall V3s \in (2^{A_{.27b}}) . \\
& \quad ((p\ (ap\ (c\_2Eprobability\_2Eprob\_space\ A_{.27a})\ V0p)) \wedge ((p\ (ap \\
& \quad (ap\ (c\_2Ebool\_2EIN\ (2^{A_{.27a}}))\ V2e)\ (ap\ (c\_2Eprobability\_2Eevents \\
& \quad A_{.27a})\ V0p))) \wedge ((\forall V4x \in A_{.27b} . ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& \quad A_{.27b})\ V4x)\ V3s)) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A_{.27a}}))\ (ap\ (ap\ ( \\
& \quad c\_2Epred\_set\_2EINTER\ A_{.27a})\ V2e)\ (ap\ V1f\ V4x)))\ (ap\ (c\_2Eprobability\_2Eevents \\
& \quad A_{.27a})\ V0p)))))) \wedge ((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A_{.27b})\ V3s)) \wedge \\
& \quad ((\forall V5x \in A_{.27b} . (\forall V6y \in A_{.27b} . ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& \quad A_{.27b})\ V5x)\ V3s)) \wedge ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A_{.27b})\ V6y)\ V3s)) \wedge \\
& \quad (\neg(V5x = V6y)))))) \Rightarrow (p\ (ap\ (ap\ (c\_2Epred\_set\_2EDISJOINT\ A_{.27a})\ ( \\
& \quad ap\ V1f\ V5x))\ (ap\ V1f\ V6y)))))) \wedge ((ap\ (ap\ (c\_2Epred\_set\_2EINTER \\
& \quad A_{.27a})\ (ap\ (c\_2Epred\_set\_2EBIGUNION\ A_{.27a})\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\
& \quad A_{.27b}\ (2^{A_{.27a}}))\ V1f)\ V3s)))\ (ap\ (c\_2Eprobability\_2Ep\_space \\
& \quad A_{.27a})\ V0p)) = (ap\ (c\_2Eprobability\_2Ep\_space\ A_{.27a})\ V0p)))))) \Rightarrow \\
& ((ap\ (ap\ (c\_2Eprobability\_2Eprob\ A_{.27a})\ V0p)\ V2e) = (ap\ (ap\ (c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE \\
& \quad A_{.27b})\ (\lambda V7x \in A_{.27b} . (ap\ (ap\ (c\_2Eprobability\_2Eprob\ A_{.27a}) \\
& \quad V0p)\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A_{.27a})\ V2e)\ (ap\ V1f\ V7x)))))) \\
& \quad V3s))))))
\end{aligned}$$