

thm_2Eprobability_2EPSPACE (TMUPnq3czxGUwusdiKVT9yHKY35Z4EWYuXr)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 6 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealax_2Ereal \quad (3)$$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Em_space A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \quad (4)$$

Definition 7 We define $c_2Eprobability_2Ep_space$ to be $\lambda A_27a : \iota.(c_2Emeasure_2Em_space A_27a)$.

Assume the following.

$$True \tag{5}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{6}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0sp \in (2^{A_27a}). (\forall V1sts \in \\ & \quad (2^{(2^{A_27a})}). (\forall V2mu \in (ty_2Erealax_2Ereal^{(2^{A_27a})}). \\ & \quad ((ap\ (c_2Emeasure_2Em_space\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (\\ & \quad 2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \\ & \quad V0sp)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}) \\ & \quad V1sts)\ V2mu)))) = V0sp)))) \end{aligned} \tag{7}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (2^{A_27a}). (\forall V1b \in \\ & \quad (2^{(2^{A_27a})}). (\forall V2c \in (ty_2Erealax_2Ereal^{(2^{A_27a})}). \\ & \quad ((ap\ (c_2Eprobability_2Ep_space\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C \\ & \quad (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \\ & \quad V0a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}) \\ & \quad V1b)\ V2c)))) = V0a)))) \end{aligned}$$