thm_2Eprobability_2Edistribution__partition (TMVAJxQCE9m7Xyxv6ksDrsYvxdajSn2DbGh)

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Let $ty\_2Eextreal\_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \tag{1}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2ET})$.

Let $c\_2Eextreal\_2Eextreal\_\_le : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{2}$$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 8 *We define* c_2Eextreal_2Eextreal__lt *to be* $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2Eextr$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{3}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{4}$$

Definition 10 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0x\in A\_27a.\lambda V1y\in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b\in((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (5)$$

Definition 11 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a:\iota.(\lambda V0x\in A\_27a.(\lambda V1f\in(2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 12 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0f\in(A\_27b^{A\_27a}).\lambda V1s\in$

Let $c\_2Emeasure\_2Esubsets:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Emeasure\_2Esubsets\ A\_27a\in((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \quad (6)$$

Definition 13 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P\in 2^{A}.$**if** $(\exists x\in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x\in A\wedge$ *of type* $\iota\Rightarrow\iota$.

Definition 14 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a:\iota.(\lambda V0P\in(2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 15 *We define* c_2Epred__set_2EBIGUNION *to be* $\lambda A\_27a:\iota.\lambda V0P\in(2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 16 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).\lambda V1t\in(2^{A\_27a}).(ap\ ($

Let $ty\_2Enum\_2Enum:\iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (7)$$

Definition 17 *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0f\in(A\_27b^{A\_27a}).\lambda V1s\in(2^{A}$

Definition 18 *We define* c_2Epred__set_2Ecountable *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).(ap$ (c_2Ebool_2E_3F

Definition 19 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1\in 2.(\lambda V1t2\in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t\in$

Definition 20 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).\lambda V1t\in(2^{A\_27a}).(ap\ (c.$

Let $c\_2Emeasure\_2Espace:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Emeasure\_2Espace\ A\_27a\in((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \quad (8)$$

Definition 21 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).\lambda V1t\in(2^{A\_27a}).(ap\ (c\_2$

Definition 22 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a:\iota.(\lambda V0x\in A_27a.$c_2Ebool_2EF).

Definition 23 *We define* c_2Emeasure_2Esubset__class *to be* $\lambda A\_27a:\iota.\lambda V0sp\in(2^{A\_27a}).\lambda V1sts\in(2^{(2^{A\_27a}}$

Definition 24 *We define* c_2Emeasure_2Ealgebra *to be* $\lambda A\_27a:\iota.\lambda V0a\in(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2}$

Definition 25 *We define* c_2Emeasure_2Esigma__algebra *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_2}$

Definition 26 *We define* c_2Epred__set_2EBIGINTER *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 27 *We define* c_2Emeasure_2Esigma *to be* $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1st \in (2^{(2^{A\_27a})}).(ap\ ($

Definition 28 *We define* c_2Emeasure_2EBorel *to be* $(ap\ (ap\ ($c_2Emeasure_2Esigma $ty\_2Eextreal\_2Eextre$

Definition 29 *We define* c_2Ecombin_2EC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 30 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

Definition 31 *We define* c_2Epred__set_2EPREIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V$

Definition 32 *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_$

Definition 33 *We define* c_2Epred__set_2EFUNSET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in ($

Definition 34 *We define* c_2Epred__set_2EPOW *to be* $\lambda A\_27a : \iota.\lambda V0set \in (2^{A\_27a}).(ap\ (c\_2Epred\_\_set\_2E$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{9}$$

Let $c\_2Emeasure\_2Em\_\_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Em\_\_space\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))}) \tag{10}$$

Let $c\_2Emeasure\_2Emeasurable\_\_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_\_sets\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))}) \tag{11}$$

Let $c\_2Emeasure\_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasure\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})} \tag{12}$$

Definition 35 *We define* c_2Emeasure_2Emeasurable *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0a \in (ty\_2Epair\_2Epro$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{13}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{14}$$

Definition 36 *We define* $\mathsf{c\_2Enum\_2E0}$ *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 37 *We define* $\mathsf{c\_2Earithmetic\_2EZERO}$ *to be* $\mathsf{c\_2Enum\_2E0}$.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{15}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{16}$$

Definition 38 *We define* $\mathsf{c\_2Enum\_2ESUC}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{17}$$

Definition 39 *We define* $\mathsf{c\_2Earithmetic\_2EBIT1}$ *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 40 *We define* $\mathsf{c\_2Earithmetic\_2ENUMERAL}$ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{18}$$

Definition 41 *We define* $\mathsf{c\_2Eprobability\_2Eprob}$ *to be* $\lambda A\_27a : \iota.(c\_2Emeasure\_2Emeasure\ A\_27a)$.

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{19}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{20}$$

Definition 42 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_REP}$ *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (t$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Eh} \tag{21}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{22}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Eh} \tag{23}$$

Definition 43 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

Definition 44 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Epred\_\_set\_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EITSET\ A\_27a\ A\_27b \in (((A\_27b^{A\_27b})^{(2^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})}) \tag{24}$$

Definition 45 *We define* c_2Ereal__sigma_2EREAL__SUM__IMAGE *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2$

Definition 46 *We define* c_2Eprobability_2Eevents *to be* $\lambda A\_27a : \iota.(c\_2Emeasure\_2Emeasurable\_\_sets\ A\_$

Definition 47 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 48 *We define* c_2Eprobability_2Ep__space *to be* $\lambda A\_27a : \iota.(c\_2Emeasure\_2Em\_\_space\ A\_27a)$.

Definition 49 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_21 (2

Definition 50 *We define* c_2Eprobability_2Edistribution *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (ty\_2Epair\_2Ep$

Let $c\_2Ereal\_2Esum : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2En} \tag{25}$$

Definition 51 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 52 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 53 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{26}$$

Definition 54 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Definition 55 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{27}$$

Definition 56 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 57 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 58 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 59 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap$ (c_2Ebool_2ECONI

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{28}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{29}$$

Definition 60 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \tag{30}$$

Let $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \tag{31}$$

Definition 61 *We define* c_2Emetric_2Emr1 *to be* $(ap\ (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal)\ (ap$ (c_

Let $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27} \tag{32}$$

Let $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \tag{33}$$

Let $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \tag{34}$$

Definition 62 *We define* c_2Emetric_2Emtop *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$

Let $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a^{A\_27b})}) \tag{35}$$

Definition 63 *We define* c_2Eseq_2E_2D_2D_3E *to be* $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

Definition 64 *We define* c_2Eseq_2Esums *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_$

Definition 65 *We define* c_2Epred__set_2EDISJOINT *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap$

Definition 66 *We define* c_2Emeasure_2Ecountably__additive *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod$

Definition 67 *We define* c_2Emeasure_2Epositive *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (t$

Definition 68 *We define* c_2Emeasure_2Emeasure__space *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A}$

Definition 69 *We define* c_2Eprobability_2Eprob__space *to be* $\lambda A\_27a : \iota.\lambda V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27}$

Definition 70 *We define* c_2Eprobability_2Erandom__variable *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0X \in (A\_27b^{A\_2}$

Assume the following.

$$True \tag{36}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{39}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{40}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{44}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P) \wedge (\forall V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V1P\ V3x))) \vee (p\ V0Q))))) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{49}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \tag{50}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A))))))) \tag{51}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{52}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow(((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{53}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in (2^{A\_27a}).(\forall V1y \in (2^{(2^{A\_27a})}).((ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0x)\ V1y)) = V0x))) \tag{54}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in (2^{A\_27a}).(\forall V1y \in (2^{(2^{A\_27a})}).((ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0x)\ V1y)) = V1y))) \tag{55}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0sp \in (2^{A\_27a}).(\forall V1sts \in (2^{(2^{A\_27a})}).(\forall V2mu \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}).((ap\ (c\_2Emeasure\_2Em\_\_space\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))\ V0sp)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))\ V1sts)\ V2mu))) = V0sp)))) \tag{56}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0sp \in (2^{A\_27a}).(\forall V1sts \in (2^{(2^{A\_27a})}).(\forall V2mu \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}).((ap\ (c\_2Emeasure\_2Emeasurable\_\_sets\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))\ V0sp)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))\ V1sts)\ V2mu))) = V1sts)))) \tag{57}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0sp \in (2^{A\_27a}).(\forall V1sts \in (2^{(2^{A\_27a})}).(\forall V2mu \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}).((ap\ (c\_2Emeasure\_2Emeasure\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))\ V0sp)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))\ V1sts)\ V2mu))) = V2mu)))) \tag{58}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0sp \in (2^{A\_27a}).(p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0sp)\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EPOW}\ A\_27a)\ V0sp))))) \tag{59}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})})).(\forall V1b \in (ty\_2Epair\_2Eprod\ (2^{A\_27b})\ (2^{(2^{A\_27b})})).(\forall V2f \in (A\_27b^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (A\_27b^{A\_27a}))\ V2f)\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasurable}\ A\_27a\ A\_27b)\ V0a)\ V1b))) \Leftrightarrow ((p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}\ A\_27a)\ V0a)) \land ((p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}\ A\_27b)\ V1b)) \land ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (A\_27b^{A\_27a}))\ V2f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFUNSET}\ A\_27a\ A\_27b)\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0a))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27b)\ V1b)))) \land (\forall V3s \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27b}))\ V3s)\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27b)\ V1b))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EPREIMAGE}\ A\_27a\ A\_27b)\ V2f)\ V3s))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0a)))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0a)))))))))))) \tag{60}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ V0s)\ V1t))\ V0s)))) \land (\forall V2s \in (2^{A\_27a}).(\forall V3t \in (2^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ V3t)\ V2s))\ V2s))))) \tag{61}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1y)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))) \Leftrightarrow (V0x = V1y)))) \tag{62}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0y \in A\_27b.(\forall V1s \in (2^{A\_27a}).(\forall V2f \in (A\_27b^{A\_27a}). ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V0y)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A\_27a.((V0y = (ap\ V2f\ V3x)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V1s)))))))) \tag{63}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1P \in (2^{A\_27a}).(\forall V2Q \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (A\_27b^{A\_27a}))\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFUNSET}\ A\_27a\ A\_27b)\ V1P)\ V2Q))) \Leftrightarrow (\forall V3x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ (ap\ V0f\ V3x))\ V2Q)))))))) \tag{64}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \Rightarrow (\forall V1f \in (A\_27b^{A\_27a}).(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V1f)\ V0s)))))) \tag{65}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0set \in (2^{A\_27a}).(\forall V1e \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ V1e)\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EPOW}\ A\_27a)\ V0set))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V1e)\ V0set))))) \tag{66}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))). (((p\ (ap\ (\mathsf{c\_2Eprobability\_2Eprob\_\_space}\ A\_27a)\ V0p)) \wedge ((\forall V1x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))\ (ap\ (\mathsf{c\_2Eprobability\_2Eevents}\ A\_27a)\ V0p))))) \wedge (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p))))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ (\lambda V2x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Eprobability\_2Eprob}\ A\_27a)\ V0p)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V2x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))))\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p)) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))) \tag{67}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))).(((p\ (ap\ (\mathsf{c\_2Eprobability\_2Eprob\_\_space}\ A\_27a)\ V0p)) \wedge ((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p))) \wedge (\forall V1x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))\ (ap\ (\mathsf{c\_2Eprobability\_2Eevents}\ A\_27a)\ V0p))))))) \Rightarrow (\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V2s)\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ V2s)\ (ap\ (\mathsf{c\_2Eprobability\_2Eevents}\ A\_27a)\ V0p))))))) \tag{68}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))).(\forall V1X \in (A\_27b^{A\_27a}).(\forall V2s \in (ty\_2Epair\_2Eprod\ (2^{A\_27b})\ (2^{(2^{A\_27b})})).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eprobability\_2Erandom\_\_variable}\ A\_27a\ A\_27b)\ V1X)\ V0p)\ V2s)) \Rightarrow (p\ (ap\ (\mathsf{c\_2Eprobability\_2Eprob\_\_space}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27b})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27b})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27b})})))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27b)\ V2s))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{A\_27b})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27b})}))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27b)\ V2s))\ (ap\ (ap\ (\mathsf{c\_2Eprobability\_2Edistribution}\ A\_27b\ A\_27a)\ V0p)\ V1X))))))))) \tag{69}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{70}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{71}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{72}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{73}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{74}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{75}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{76}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{77}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{78}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{79}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{80}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{81}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))).\ (\forall V1X \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).(((p\ (ap\ (\mathsf{c\_2Eprobability\_2Eprob\_\_space}\ A\_27a)\ V0p)) \wedge ((\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V2x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))\ (ap\ (\mathsf{c\_2Eprobability\_2Eevents}\ A\_27a)\ V0p))))) \wedge ((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p))) \wedge (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eprobability\_2Erandom\_\_variable}\ A\_27a\ ty\_2Eextreal\_2Eextreal)\ V1X)\ V0p)\ \mathsf{c\_2Emeasure\_2EBorel}))))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ ty\_2Eextreal\_2Eextreal)\ (\lambda V3x \in ty\_2Eextreal\_2Eextreal.(ap\ (ap\ (ap\ (\mathsf{c\_2Eprobability\_2Edistribution}\ ty\_2Eextreal\_2Eextreal\ A\_27a)\ V0p)\ V1X)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Eextreal\_2Eextreal)\ V3x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Eextreal\_2Eextreal)))))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ ty\_2Eextreal\_2Eextreal)\ V1X)\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p))) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))))$$