

thm_2Eprobability_2Efinite__marginal__product__space__POW3 (TMKLSCAaX6PifNY7eP24Hfzb6bmph1ce75r)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2EBOUNDED$ to be $(\lambda V0v \in 2.c_2Ebool_2E_ET)$.

Definition 5 We define $c_2Ecombin_2E_ES$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.(\lambda V0f \in ((A.\lambda 27c^{A-27b})^{A-27a}))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 7 We define $c_2Ecombin_2E_Eo$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.(\lambda V0f \in (A.\lambda 27b^{A-27c}).\lambda V1g$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty\ A.\lambda 27a \Rightarrow c_2Emeasure_2Emeasure\ A.\lambda 27a \in ((ty_2Erealax_2Ereal^{(2^{A-27a})})(ty_2Epair_2Eprod\ (2^{A-27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A-27a})}))\ (ty_2Erealax_2Ereal^{(2^{A-27a})}))) \tag{3}$$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty\ A.\lambda 27a \Rightarrow c_2Emeasure_2Emeasurable_sets\ A.\lambda 27a \in (((2^{(2^{A-27a})})(ty_2Epair_2Eprod\ (2^{A-27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A-27a})}))\ (ty_2Erealax_2Ereal^{(2^{A-27a})})))) \tag{4}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (5)$$

Definition 8 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2ET)$.

Definition 9 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ V2t))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (6)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2EABS_prod\ V0x\ V1y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (7)$$

Definition 13 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (A_27b^{A_27a}). (ap\ (c_2Epred_set_2EGSPEC\ V0f\ V1s))$

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ V0P))))$

Definition 15 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap\ (c_2Epred_set_2EGSPEC\ V0P))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (9)$$

Definition 16 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (10)$$

Definition 17 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2. V0t))$.

Definition 18 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (12)$$

Definition 19 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 20 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 21 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 22 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2$

Definition 23 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (13)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal_REP_CLASS}) \quad (14)$$

Definition 24 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (t$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (15)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (16)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (17)$$

Definition 25 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 26 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (18)$$

Definition 27 We define $c_2Erealx_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal$

Definition 28 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealx_2Ereal.\lambda V1y \in ty_2Erealx_2Ereal$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}) \quad (19)$$

Let $c_2Erealx_2Etrealt : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (20)$$

Definition 29 We define $c_2Erealx_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal$

Definition 30 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealx_2Ereal.\lambda V1y \in ty_2Erealx_2Ereal$

Definition 31 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Definition 32 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealx_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (21)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (22)$$

Definition 33 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (23)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric \\ A_27a)^{(ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \end{aligned} \quad (24)$$

Definition 34 We define $c_2Emetric_2Emr1$ to be $(ap\ (c_2Emetric_2Emetric\ ty_2Erealx_2Ereal)\ (ap\ (c_2Emetric_2Emetric$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}) \quad (25)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (26)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^A-27a)})}) \quad (27)$$

Definition 35 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap$

Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Etends\ A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ (2^{A-27b})^{A-27b})})_{A_27a})_{(A_27a^{A-27b})}) \quad (28)$$

Definition 36 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 37 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty$

Definition 38 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF).$

Definition 39 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c$

Definition 40 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap$

Definition 41 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A-27a}).\lambda V1Q \in ($

Definition 42 We define $c_2Emeasure_2Ecountably_additive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 43 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A-27a})\ (ty$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in ((2^{A-27a})_{(ty_2Epair_2Eprod\ (2^{A-27a})\ (ty_2Epair_2Eprod\ (2^{(2^A-27a)})\ (ty_2Erealax_2Ereal^{(2^A-27a)}))}) \quad (29)$$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in ((2^{(2^A-27a)})_{(ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^A-27a)})}) \quad (30)$$

Definition 44 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ ($

Definition 45 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A-27a}).\lambda V1s \in (2^A$

Definition 46 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap\ (c_2Ebool_2E3F$

Definition 47 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$
 Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Espace A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))}) \quad (31)$$

Definition 48 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 49 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})})$

Definition 50 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))$

Definition 51 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))$

Definition 52 We define $c_2Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))$

Definition 53 We define $c_2Emeasure_2Eadditive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))$

Definition 54 We define $c_2Epred_set_2ECROSS$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A_27b})$

Definition 55 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2E$

Definition 56 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E21 (2^{A_27a}))$

Definition 57 We define $c_2Epred_set_2EPOW$ to be $\lambda A_27a : \iota.\lambda V0set \in (2^{A_27a}).(ap (c_2Epred_set_2E2E (2^{A_27a}))$

Definition 58 We define $c_2Epred_set_2EPREIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1g \in (A_27a^{A_27b})$

Definition 59 We define $c_2Emeasure_2Emeasurable$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))$

Definition 60 We define $c_2Eprobability_2Eevents$ to be $\lambda A_27a : \iota.(c_2Emeasure_2Emeasurable_sets A_27a)$

Definition 61 We define $c_2Eprobability_2Eprob_space$ to be $\lambda A_27a : \iota.(c_2Emeasure_2Em_space A_27a)$.

Definition 62 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (32)$$

Definition 63 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B (c_2Earithmetic_2E_2B)))$

Definition 64 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 65 We define $c_2Eprobability_2Eprob_space$ to be $\lambda A_27a : \iota.\lambda V0p \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))$

Definition 66 We define $c_2Eprobability_2Erandom_variable$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0X \in (A_27b^{A_27a})$

Definition 67 We define $c_2Eprobability_2Eprob$ to be $\lambda A_27a : \iota.(c_2Emeasure_2Emeasure A_27a)$.

Definition 68 We define $c_2Eprobability_2Ejoint_distribution3$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d$

Assume the following.

$$True \quad (33)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (39)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (43)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\neg(\exists V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (44)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p V0P) \wedge (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p V0P) \wedge (p (ap V1Q V3x))))))) \quad (45)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).((\forall V2x \in A.27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a.(p (ap V1Q V3x))))))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \quad (47)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee ((p V1B) \wedge (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (48)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (49)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (50)$$

Assume the following.

$$(\forall V0v \in 2.((p (ap c.2Ebool.2EBOUNDED V0v)) \Leftrightarrow True)) \quad (51)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in (2^{A.27a}).(\forall V1y \in (2^{(2^{A.27a})}).((ap (c.2Emeasure.2Espace A.27a) (ap (ap (c.2Epair.2E.2C (2^{A.27a}) (2^{(2^{A.27a})})) V0x) V1y)) = V0x))) \quad (52)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in (2^{A.27a}). (\forall V1y \in \\ & (2^{(2^{A.27a})}). ((ap\ (c.2Emeasure_2Esubsets\ A.27a)\ (ap\ (ap\ (c.2Epair_2E_2C \\ & (2^{A.27a})\ (2^{(2^{A.27a})}))\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0sp \in (2^{A.27a}). (\forall V1sts \in \\ & (2^{(2^{A.27a})}). (\forall V2mu \in (ty_2Erealax_2Ereal^{(2^{A.27a})}). \\ & ((ap\ (c.2Emeasure_2Emeasurable_sets\ A.27a)\ (ap\ (ap\ (c.2Epair_2E_2C \\ & (2^{A.27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A.27a})})\ (ty_2Erealax_2Ereal^{(2^{A.27a})}))) \\ & V0sp)\ (ap\ (ap\ (c.2Epair_2E_2C\ (2^{(2^{A.27a})})\ (ty_2Erealax_2Ereal^{(2^{A.27a})})) \\ & V1sts)\ V2mu))) = V1sts)))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0sp \in (2^{A.27a}). (\forall V1sts \in \\ & (2^{(2^{A.27a})}). (\forall V2mu \in (ty_2Erealax_2Ereal^{(2^{A.27a})}). \\ & ((ap\ (c.2Emeasure_2Emeasure\ A.27a)\ (ap\ (ap\ (c.2Epair_2E_2C\ (2^{A.27a}) \\ & (ty_2Epair_2Eprod\ (2^{(2^{A.27a})})\ (ty_2Erealax_2Ereal^{(2^{A.27a})}))) \\ & V0sp)\ (ap\ (ap\ (c.2Epair_2E_2C\ (2^{(2^{A.27a})})\ (ty_2Erealax_2Ereal^{(2^{A.27a})})) \\ & V1sts)\ V2mu))) = V2mu)))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0sp \in (2^{A.27a}). (p\ (ap \\ & (c.2Emeasure_2Esigma_algebra\ A.27a)\ (ap\ (ap\ (c.2Epair_2E_2C \\ & (2^{A.27a})\ (2^{(2^{A.27a})}))\ V0sp)\ (ap\ (c.2Epred_set_2EPOW\ A.27a) \\ & V0sp)))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (ty_2Epair_2Eprod \\
& \quad (2^{A_27a})\ (2^{(2^{A_27a})})). (\forall V1m \in (ty_2Erealax_2Ereal^{(2^{A_27a})}). \\
& \quad (((p\ (ap\ (c_2Emeasure_2Esigma_algebra\ A_27a)\ V0s)) \wedge ((p\ (ap\ (\\
& \quad c_2Epred_set_2EFINITE\ A_27a)\ (ap\ (c_2Emeasure_2Espace\ A_27a) \\
& \quad V0s))) \wedge ((p\ (ap\ (c_2Emeasure_2Epositive\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))) \\
& \quad (ap\ (c_2Emeasure_2Espace\ A_27a)\ V0s))\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))\ (ap\ (c_2Emeasure_2Esubsets \\
& \quad A_27a)\ V0s))\ V1m)))) \wedge (p\ (ap\ (c_2Emeasure_2Eadditive\ A_27a)\ (ap \\
& \quad (ap\ (c_2Epair_2E_2C\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})}) \\
& \quad (ty_2Erealax_2Ereal^{(2^{A_27a})}))))\ (ap\ (c_2Emeasure_2Espace\ A_27a) \\
& \quad V0s))\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))) \\
& \quad (ap\ (c_2Emeasure_2Esubsets\ A_27a)\ V0s))\ V1m)))))) \Rightarrow (p\ (ap\ (c_2Emeasure_2Emeasure_space \\
& \quad A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})}) \\
& \quad (ty_2Erealax_2Ereal^{(2^{A_27a})})))\ (ap\ (c_2Emeasure_2Espace\ A_27a) \\
& \quad V0s))\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))) \\
& \quad (ap\ (c_2Emeasure_2Esubsets\ A_27a)\ V0s))\ V1m)))))) \\
& \hspace{15em} (57)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\
& \quad A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\
& \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\
& \hspace{15em} (58)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& \quad (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))) \\
& \hspace{15em} (59)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& \quad (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& \quad V2x)\ (ap\ (ap\ (c_2Epred_set_2EUNION\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\
& \quad (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A_27a)\ V2x)\ V1t)))))) \\
& \hspace{15em} (60)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& \quad (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& \quad V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\
& \quad (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A_27a)\ V2x)\ V1t)))))) \\
& \hspace{15em} (61)
\end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & ((\forall V0s \in (2^{A.27a}).((ap\ (\\ & ap\ (c.2Epred_set_2EINTER\ A.27a)\ (c.2Epred_set_2EEMPTY\ A.27a)) \\ & V0s) = (c.2Epred_set_2EEMPTY\ A.27a))) \wedge (\forall V1s \in (2^{A.27a}). \\ & ((ap\ (ap\ (c.2Epred_set_2EINTER\ A.27a)\ V1s)\ (c.2Epred_set_2EEMPTY \\ & A.27a)) = (c.2Epred_set_2EEMPTY\ A.27a)))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ & (2^{A.27a}).((p\ (ap\ (ap\ (c.2Epred_set_2EDISJOINT\ A.27a)\ V0s)\ V1t)) \Leftrightarrow \\ & (\neg(\exists V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a)\ V2x)\ V0s)) \wedge \\ & (p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a)\ V2x)\ V1t)))))))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27b}).(\forall V2x \in \\ & (ty_2Epair_2Eprod\ A.27a\ A.27b).((p\ (ap\ (ap\ (c.2Ebool_2EIN\ ty_2Epair_2Eprod \\ & A.27a\ A.27b))\ V2x)\ (ap\ (ap\ (c.2Epred_set_2ECROSS\ A.27a\ A.27b) \\ & V0P)\ V1Q))) \Leftrightarrow ((p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a)\ (ap\ (c.2Epair_2EFST \\ & A.27a\ A.27b)\ V2x))\ V0P)) \wedge (p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27b)\ (ap\ (c.2Epair_2ESND \\ & A.27a\ A.27b)\ V2x))\ V1Q)))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27b}).(((p\ (ap\ (c.2Epred_set_2EFINITE \\ & A.27a)\ V0P)) \wedge (p\ (ap\ (c.2Epred_set_2EFINITE\ A.27b)\ V1Q))) \Rightarrow (p \\ & (ap\ (c.2Epred_set_2EFINITE\ (ty_2Epair_2Eprod\ A.27a\ A.27b)) \\ & (ap\ (ap\ (c.2Epred_set_2ECROSS\ A.27a\ A.27b)\ V0P)\ V1Q)))))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0set \in (2^{A.27a}).(\forall V1e \in \\ & (2^{A.27a}).((p\ (ap\ (ap\ (c.2Ebool_2EIN\ (2^{A.27a}))\ V1e)\ (ap\ (c.2Epred_set_2EPOW \\ & A.27a)\ V0set))) \Leftrightarrow (p\ (ap\ (ap\ (c.2Epred_set_2ESUBSET\ A.27a)\ V1e) \\ & V0set)))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0f \in (A.27b^{A.27a}).(\forall V1s \in (2^{A.27b}).(\forall V2x \in \\ & A.27a.((p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a)\ V2x)\ (ap\ (ap\ (c.2Epred_set_2EPREIMAGE \\ & A.27a\ A.27b)\ V0f)\ V1s))) \Leftrightarrow (p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27b)\ (ap\ V0f \\ & V2x))\ V1s)))))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Epred_set_2EPREIMAGE \\ & \quad A_27a\ A_27b)\ V0f)\ (c_2Epred_set_2EEMPTY\ A_27b)) = (c_2Epred_set_2EEMPTY \\ & \quad A_27a))) \end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\ & \quad (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))). \\ & \quad ((p\ (ap\ (c_2Eprobability_2Eprob_space\ A_27a)\ V0p)) \Rightarrow ((ap\ (ap \\ & \quad (c_2Eprobability_2Eprob\ A_27a)\ V0p)\ (c_2Epred_set_2EEMPTY \\ & \quad A_27a)) = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)))) \end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\ & \quad (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))). \\ & \quad (\forall V1s \in (2^{A_27a}).((p\ (ap\ (c_2Eprobability_2Eprob_space \\ & \quad A_27a)\ V0p)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ V1s)\ (ap\ (c_2Eprobability_2Eevents \\ & \quad A_27a)\ V0p)))) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Ereal_2Ereal_of_num \\ & \quad c_2Enum_2E0))\ (ap\ (ap\ (c_2Eprobability_2Eprob\ A_27a)\ V0p)\ V1s)))))) \end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\ & \quad (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))). \\ & \quad (\forall V1s \in (2^{A_27a}).(\forall V2t \in (2^{A_27a}).(\forall V3u \in \\ & \quad (2^{A_27a}).(((p\ (ap\ (c_2Eprobability_2Eprob_space\ A_27a)\ V0p)) \wedge \\ & \quad ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ V1s)\ (ap\ (c_2Eprobability_2Eevents \\ & \quad A_27a)\ V0p)))) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ V2t)\ (ap\ (c_2Eprobability_2Eevents \\ & \quad A_27a)\ V0p)))) \wedge ((p\ (ap\ (ap\ (c_2Epred_set_2EDISJOINT\ A_27a)\ V1s) \\ & \quad V2t)) \wedge (V3u = (ap\ (ap\ (c_2Epred_set_2EUNION\ A_27a)\ V1s)\ V2t)))))) \Rightarrow \\ & \quad ((ap\ (ap\ (c_2Eprobability_2Eprob\ A_27a)\ V0p)\ V3u) = (ap\ (ap\ c_2Erealax_2Ereal_add \\ & \quad (ap\ (ap\ (c_2Eprobability_2Eprob\ A_27a)\ V0p)\ V1s)\ (ap\ (ap\ (c_2Eprobability_2Eprob \\ & \quad A_27a)\ V0p)\ V2t)))))) \end{aligned} \tag{71}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{72}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{73}$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \tag{74}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (75)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (76)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (77)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (78)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (79)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (80)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (81)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (82)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (83)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (84)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (85)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (86)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\ & \text{nonempty } A_27c \Rightarrow \forall A_27d.\text{nonempty } A_27d \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\ & (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal(2^{A_27a}))))). \\ & (\forall V1s1 \in (2^{A_27b}).(\forall V2s2 \in (2^{A_27c}).(\forall V3s3 \in \\ & (2^{A_27d}).(\forall V4X \in (A_27b^{A_27a}).(\forall V5Y \in (A_27c^{A_27a}). \\ & (\forall V6Z \in (A_27d^{A_27a}).((((ap (c_2Epred_set_2EPOW A_27a) \\ & (ap (c_2Eprobability_2Ep_space A_27a) V0p)) = (ap (c_2Eprobability_2Eevents \\ & A_27a) V0p)) \wedge ((p (ap (ap (ap (c_2Eprobability_2Erandom_variable \\ & A_27a A_27b) V4X) V0p) (ap (ap (c_2Epair_2E_2C (2^{A_27b}) (2^{(2^{A_27b})}) \\ & V1s1) (ap (c_2Epred_set_2EPOW A_27b) V1s1)))) \wedge ((p (ap (ap (ap \\ & (c_2Eprobability_2Erandom_variable A_27a A_27c) V5Y) V0p) (\\ & ap (ap (c_2Epair_2E_2C (2^{A_27c}) (2^{(2^{A_27c})}) V2s2) (ap (c_2Epred_set_2EPOW \\ & A_27c) V2s2)))) \wedge ((p (ap (ap (ap (c_2Eprobability_2Erandom_variable \\ & A_27a A_27d) V6Z) V0p) (ap (ap (c_2Epair_2E_2C (2^{A_27d}) (2^{(2^{A_27d})}) \\ & V3s3) (ap (c_2Epred_set_2EPOW A_27d) V3s3)))) \wedge ((p (ap (c_2Epred_set_2EFINITE \\ & A_27a) (ap (c_2Eprobability_2Ep_space A_27a) V0p)) \wedge ((p (ap \\ & (c_2Epred_set_2EFINITE A_27b) V1s1)) \wedge ((p (ap (c_2Epred_set_2EFINITE \\ & A_27c) V2s2)) \wedge (p (ap (c_2Epred_set_2EFINITE A_27d) V3s3))))))))) \Rightarrow \\ & (p (ap (c_2Emeasure_2Emeasure_space (ty_2Epair_2Eprod A_27b \\ & (ty_2Epair_2Eprod A_27c A_27d))) (ap (ap (c_2Epair_2E_2C (2^{(ty_2Epair_2Eprod A_27b (ty_2Epair_2Eprod A_27c \\ & A_27d))}) \\ & (ty_2Epair_2Eprod (2^{(2^{(ty_2Epair_2Eprod A_27b (ty_2Epair_2Eprod A_27c A_27d))})}) \\ & (ty_2Erealax_2Ereal(2^{(ty_2Epair_2Eprod A_27b (ty_2Epair_2Eprod A_27c A_27d))})))))) \\ & (ap (ap (c_2Epred_set_2ECROSS A_27b (ty_2Epair_2Eprod A_27c \\ & A_27d)) V1s1) (ap (ap (c_2Epred_set_2ECROSS A_27c A_27d) V2s2) \\ & V3s3))) (ap (ap (c_2Epair_2E_2C (2^{(ty_2Epair_2Eprod A_27b (ty_2Epair_2Eprod A_27c A_27d))}) \\ & (ty_2Erealax_2Ereal(2^{(ty_2Epair_2Eprod A_27b (ty_2Epair_2Eprod A_27c A_27d))})))))) \\ & (ap (c_2Epred_set_2EPOW (ty_2Epair_2Eprod A_27b (ty_2Epair_2Eprod \\ & A_27c A_27d))) (ap (ap (c_2Epred_set_2ECROSS A_27b (ty_2Epair_2Eprod \\ & A_27c A_27d)) V1s1) (ap (ap (c_2Epred_set_2ECROSS A_27c A_27d) \\ & V2s2) V3s3))) (ap (ap (ap (ap (c_2Eprobability_2Ejoint_distribution3 \\ & A_27b A_27c A_27d A_27a) V0p) V4X) V5Y) V6Z))))))))))))) \end{aligned}$$