

thm_2Eprobability_2Efinite__support__expectation (TMbXo9qFo9TwDr8aTQ8SykzoiDZAcVCxoUn)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 4 We define $c_2Ecombin_2EC$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a})).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 6 We define $c_2Ecombin_2Eo$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealx_2Ereal^{(2^{A_27a})})))}) \tag{3}$$

Definition 7 We define $c_2Eprobability_2Ep_space$ to be $\lambda A.\lambda A_27a : \iota.(c_2Emeasure_2Em_space\ A_27a)$.

Definition 8 We define c_2Ebool_2EIN to be $\lambda A.\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a})).(ap\ V1f\ V0x))$

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{10}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{11}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{12}$$

Definition 20 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \tag{13}$$

Definition 21 We define $c_2Eextreal_2Eextreal_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Eextrealax_2Eextreal\ n)$.

Let $c_2Eextreal_2Eextreal_add : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_add \in ((ty_2Eextrealax_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \tag{14}$$

Let $c_2Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EITSET\ A_27a\ A_27b \in (((A_27b^{A_27b})^{(2^{A_27a})})^{((A_27b^{A_27b})^{A_27a})}) \tag{15}$$

Definition 22 We define $c_2Eextreal_2EEXTREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextrealax_2Eextreal\ A_27a)$.

Let $c_2Eextreal_2Eextreal_ainv : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_ainv \in (ty_2Eextrealax_2Eextreal^{ty_2Eextreal_2Eextreal}) \tag{16}$$

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((ty_2Eextrealax_2Eextreal)^{ty_2Eextreal_2Eextreal}) \tag{17}$$

Definition 23 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E7E\ t))$.

Definition 24 We define $c_2Eextreal_2Eextreal_lt$ to be $\lambda V0x \in ty_2Eextrealax_2Eextreal.\lambda V1y \in ty_2Eextrealax_2Eextreal.$

Definition 25 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 26 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ c_2Emin_2E40\ t1\ t2))))$.

Definition 27 We define $c_2Emeasure_2Efn_minus$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextrealax_2Eextreal^{A_27a})$.

Definition 28 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A-27a}).\lambda V1s \in$

Definition 29 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 30 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap\ (c_2Epred_set_2E$

Definition 31 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (18)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal_REP_CLASS}) \quad (19)$$

Definition 32 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (ty_2Erealax_2Ereal_REP$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (20)$$

Definition 33 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 34 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 35 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap\ (c_2Ebool_2E_21\ (2^{A-27a}$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets\ A_27a \in ((2^{(2^{A-27a})})^{(ty_2Epair_2Eprod\ (2^{A-27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A-27a})})\ (ty_2Erealax_2Ereal^{(2^{A-27a})})))}) \quad (21)$$

Definition 36 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \quad (22)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \quad (23)$$

Definition 37 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Enum_2ESUC_REP$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (24)$$

Definition 38 We define $c_2Earithmic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic$

Definition 39 We define $c_2Earithmic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 40 We define $c_2Emeasure_2Eindicator_fn$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(\lambda V1x \in A_27a.(ap$

Definition 41 We define $c_2Emeasure_2Epos_simple_fn$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^A$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND \\ A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \end{aligned} \quad (25)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST \\ A_27a A_27b \in (A_27a)^{(ty_2Epair_2Eprod A_27a A_27b)} \end{aligned} \quad (26)$$

Definition 42 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a}$

Definition 43 We define $c_2ELebesgue_2Epsfs$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2E$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)} \end{aligned} \quad (27)$$

Let $c_2Erealax_2Etrealeq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealeq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \quad (28)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})} \quad (29)$$

Definition 44 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty$

Definition 45 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Erealax_2Ereal_add : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Ereal_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)} \end{aligned} \quad (30)$$

Definition 46 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Definition 47 We define $c_2Ereal_sigma_2EREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2$

Definition 48 We define $c_2E\text{lebesgue_2Epos_simple_fn_integral}$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2E\text{pair_2E} \dots$

Definition 49 We define $c_2E\text{lebesgue_2Epsfis}$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2E\text{pair_2Eprod } (2^{A-27a}) (ty_2E$

Definition 50 We define $c_2E\text{real_2Esup}$ to be $\lambda V0P \in (2^{ty_2E\text{realax_2Ereal}}).(ap (c_2E\text{min_2E} \dots$

Let $c_2E\text{extreal_2ENegInf} : \iota$ be given. Assume the following.

$$c_2E\text{extreal_2ENegInf} \in ty_2E\text{extreal_2Eextreal} \quad (31)$$

Let $c_2E\text{extreal_2EPosInf} : \iota$ be given. Assume the following.

$$c_2E\text{extreal_2EPosInf} \in ty_2E\text{extreal_2Eextreal} \quad (32)$$

Definition 51 We define $c_2E\text{extreal_2Eextreal_sup}$ to be $\lambda V0p \in (2^{ty_2E\text{extreal_2Eextreal}}).(ap (ap (ap (c_2E$

Definition 52 We define $c_2E\text{lebesgue_2Epos_fn_integral}$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2E\text{pair_2Eprod } (2^{A-27a}) (ty_2E$

Definition 53 We define $c_2E\text{measure_2Efn_plus}$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2E\text{extreal_2Eextreal}^{A-27a}).$

Let $c_2E\text{extreal_2Eextreal_sub} : \iota$ be given. Assume the following.

$$c_2E\text{extreal_2Eextreal_sub} \in ((ty_2E\text{extreal_2Eextreal}^{ty_2E\text{extreal_2Eextreal}})_{ty_2E\text{extreal_2Eextreal}}) \quad (33)$$

Definition 54 We define $c_2E\text{lebesgue_2Eintegral}$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2E\text{pair_2Eprod } (2^{A-27a}) (ty_2E$

Definition 55 We define $c_2E\text{probability_2Eexpectation}$ to be $\lambda A_27a : \iota.(c_2E\text{lebesgue_2Eintegral } A_27a).$

Definition 56 We define $c_2E\text{pred_set_2EUNIV}$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2E\text{bool_2E} \dots$

Let $c_2E\text{measure_2Esubsets} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2E\text{measure_2Esubsets } A_27a \in (\quad (34)$$

Definition 57 We define $c_2E\text{pred_set_2ESUBSET}$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap ($

Definition 58 We define $c_2E\text{pred_set_2EINJ}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A-27a}).\lambda V1s \in (2^{A-27a}).$

Definition 59 We define $c_2E\text{pred_set_2Ecountable}$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c_2E\text{bool_2E} \dots$

Definition 60 We define $c_2E\text{pred_set_2EUNION}$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c_2E$

Let $c_2E\text{measure_2Espace} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2E\text{measure_2Espace } A_27a \in ((2^{A-27a})_{(ty_2E\text{pair_2Eprod } (2^{A-27a}) (2^{(2^{A-27a})}))}) \quad (35)$$

Definition 61 We define $c_2E\text{pred_set_2EDIFF}$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c_2E$

Definition 62 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A-27a}).\lambda V1sts \in (2^{(2^{A-27a})})$

Definition 63 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A-27a})) (2^{(2^{A-27a})})$

Definition 64 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A-27a})) (2^{(2^{A-27a})})$

Definition 65 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap (c_2Epred_set_2EBIGINTER) (2^{(2^{A-27a})}))$

Definition 66 We define $c_2Emeasure_2Esigma$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A-27a}).\lambda V1st \in (2^{(2^{A-27a})}).(ap (c_2Emeasure_2Esigma) (2^{(2^{A-27a})}))$

Definition 67 We define $c_2Emeasure_2EBorel$ to be $(ap (ap (c_2Emeasure_2Esigma) ty_2Eextreal_2Eextreal) (2^{(2^{A-27a})}))$

Definition 68 We define $c_2Eprobability_2Eevents$ to be $\lambda A_27a : \iota.(c_2Emeasure_2Emeasurable_sets A_27a) (2^{(2^{A-27a})})$

Definition 69 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A-27a}).\lambda V1Q \in (2^{(2^{A-27a})})$

Definition 70 We define $c_2Emeasure_2Emeasurable$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A-27a})) (2^{(2^{A-27a})})$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum})})^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum)}) (36)$$

Definition 71 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Definition 72 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Definition 73 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) (37)$$

Definition 74 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal (c_2Erealax_2Ereal_neg))$

Definition 75 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 76 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECONJ) (c_2Ereal_2Ereal_sub)) (c_2Ereal_2Ereal_neg)) (c_2Ereal_2Ereal))$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Emetric_2Emetric A0) (38)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Emetric A_27a \in ((ty_2Emetric_2Emetric A_27a)^{(ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod A_27a A_27a)})}) (39)$$

Definition 77 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric ty_2Erealx_2Ereal) (ap (c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Edist A_27a \in ((ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod A_27a A_27b)})_{A_27a}) \quad (40)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (41)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^A_27a)}})) \quad (42)$$

Definition 78 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric A_27a).(ap (c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Etends A_27a A_27b \in (((2^{(ty_2Epair_2Eprod (ty_2Etopology_2Etopology A_27a) ((2^{A_27b})^{A_27b}))})_{A_27a})_{(A_27a)^{A_27b}})) \quad (43)$$

Definition 79 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1x \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2Enum_2Enum$

Definition 80 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2Enum_2Enum$

Definition 81 We define $c_2Emeasure_2Ecountably_additive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}))$

Definition 82 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}))$

Definition 83 We define $c_2Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}))$

Definition 84 We define $c_2Eprobability_2Eprob_space$ to be $\lambda A_27a : \iota.\lambda V0p \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}))$

Definition 85 We define $c_2Eprobability_2Ereal_random_variable$ to be $\lambda A_27a : \iota.\lambda V0X \in (ty_2Eextrealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2Enum_2Enum$

Assume the following.

$$True \quad (44)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (45)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (47)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (48)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (49)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (50)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (51)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (52)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (53)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\
& (2^{A.27a}) (ty_2Epair_2Eprod (2^{(2^{A.27a})}) (ty_2Erealax_2Ereal^{(2^{A.27a})}))). \\
& (\forall V1X \in (ty_2Eextreal_2Eextreal^{A.27a}).(((p (ap (c_2Epred_set_2EFINITE \\
& ty_2Eextreal_2Eextreal) (ap (ap (c_2Epred_set_2EIMAGE\ A.27a \\
& ty_2Eextreal_2Eextreal) V1X) (ap (c_2Eprobability_2Ep_space \\
& A.27a) V0p)))) \wedge (p (ap (ap (c_2Eprobability_2Ereal_random_variable \\
& A.27a) V1X) V0p))) \Rightarrow ((ap (ap (c_2Eprobability_2Eexpectation\ A.27a) \\
& V0p) V1X) = (ap (ap (c_2Eextreal_2EEXTREAL_SUM_IMAGE\ ty_2Eextreal_2Eextreal) \\
& (\lambda V2r \in ty_2Eextreal_2Eextreal.(ap (ap\ c.2Eextreal_2Eextreal_mul \\
& V2r) (ap\ c.2Eextreal_2ENormal) (ap (ap (c_2Eprobability_2Eprob \\
& A.27a) V0p) (ap (ap (c_2Epred_set_2EINTER\ A.27a) (ap (ap (c_2Epred_set_2EPREIMAGE \\
& A.27a\ ty_2Eextreal_2Eextreal) V1X) (ap (ap (c_2Epred_set_2EINSERT \\
& ty_2Eextreal_2Eextreal) V2r) (c_2Epred_set_2EEMPTY\ ty_2Eextreal_2Eextreal)))) \\
& (ap (c_2Eprobability_2Ep_space\ A.27a) V0p)))))) (ap (ap (c_2Epred_set_2EIMAGE \\
& A.27a\ ty_2Eextreal_2Eextreal) V1X) (ap (c_2Eprobability_2Ep_space \\
& A.27a) V0p))))))
\end{aligned} \tag{54}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{55}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{58}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{64}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{65}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{66}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\
& (2^{A.27a}) (ty_2Epair_2Eprod (2^{(2^{A.27a})}) (ty_2Erealax_2Ereal^{(2^{A.27a})}))). \\
& (\forall V1X \in (ty_2Eextreal_2Eextreal^{A.27a}). (((p (ap (c.2Epred_set_2EFINITE \\
& ty_2Eextreal_2Eextreal) (ap (ap (c.2Epred_set_2EIMAGE A.27a \\
& ty_2Eextreal_2Eextreal) V1X) (ap (c.2Eprobability_2Ep_space \\
& A.27a) V0p)))) \wedge (p (ap (ap (c.2Eprobability_2Ereal_random_variable \\
& A.27a) V1X) V0p))) \Rightarrow ((ap (ap (c.2Eprobability_2Eexpectation A.27a) \\
& V0p) V1X) = (ap (ap (c.2Eextreal_2EEXTREAL_SUM_IMAGE ty_2Eextreal_2Eextreal) \\
& (\lambda V2r \in ty_2Eextreal_2Eextreal. (ap (ap c.2Eextreal_2Eextreal_mul \\
& V2r) (ap c.2Eextreal_2ENormal (ap (ap (ap (c.2Eprobability_2Edistribution \\
& ty_2Eextreal_2Eextreal A.27a) V0p) V1X) (ap (ap (c.2Epred_set_2EINSERT \\
& ty_2Eextreal_2Eextreal) V2r) (c.2Epred_set_2EEMPTY ty_2Eextreal_2Eextreal)))))) \\
& (ap (ap (c.2Epred_set_2EIMAGE A.27a ty_2Eextreal_2Eextreal) \\
& V1X) (ap (c.2Eprobability_2Ep_space A.27a) V0p))))))
\end{aligned}$$