

thm_2Eprobability_2Ejoint_distribution_le2
(TMdymP2NTPz6EAXwQgy7jd4vCNW2nJm3wbC)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (V0P)) (V1P)))$

Definition 4 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_EF))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \tag{2}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a)^{(ty_2Epair_2Eprod A_27a A_27b)} \tag{3}$$

Definition 7 We define $c_2Ebool_2E_IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (4)$$

Definition 9 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (5)$$

Definition 10 We define $c_2Epred_set_2ECROSS$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A$

Definition 11 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ ($

Definition 12 We define $c_2Epred_set_2EPOW$ to be $\lambda A_27a : \iota.\lambda V0set \in (2^{A_27a}).(ap\ (c_2Epred_set_2E$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (6)$$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in \\ ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \end{aligned} \quad (7)$$

Definition 13 We define $c_2Eprobability_2Ep_space$ to be $\lambda A_27a : \iota.(c_2Emeasure_2Em_space\ A_27a).$

Definition 14 We define $c_2Epred_set_2EPREIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b)^{A_27a}.\lambda V$

Definition 15 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasure\ A_27a \in (\\ (ty_2Erealax_2Ereal^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))) \end{aligned} \quad (8)$$

Definition 16 We define $c_2Eprobability_2Eprob$ to be $\lambda A_27a : \iota.(c_2Emeasure_2Emeasure\ A_27a).$

Definition 17 We define $c_2Eprobability_2Edistribution$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (ty_2Epair_2E$

Definition 18 We define $c_2Eprobability_2Ejoint_distribution$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (9)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (10)$$

Definition 19 We define $c_2Emin_2E.40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p\ x)$ of type $\iota \Rightarrow \iota$).

Definition 20 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E.40\ (t$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (11)$$

Definition 21 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 22 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets \\ A_27a \in & ((2^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \end{aligned} \quad (12)$$

Definition 23 We define $c_2Eprobability_2Eevents$ to be $\lambda A_27a : \iota.(c_2Emeasure_2Emeasurable_sets\ A_27a)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (13)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (14)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (15)$$

Definition 24 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 25 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (17)$$

Definition 26 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$
Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 27 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 28 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (19)$$

Definition 29 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 30 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 31 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 32 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap\ (c_2Epred_s$

Definition 33 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (20)$$

Definition 34 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 35 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 36 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 37 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (21)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (22)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})}) \quad (23)$$

Definition 38 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 39 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal)$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (24)$$

Definition 40 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 41 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 42 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 43 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Definition 44 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a}$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (25)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric\ A_27a)^{(ty_2Erealax_2Ereal)^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}})) \quad (26)$$

Definition 45 We define $c_2Emetric_2Emr1$ to be $(ap\ (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal)\ (ap\ (c_2Emetric_2Emetric$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal)^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}) \quad (27)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (28)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^{A_27a})})}) \quad (29)$$

Definition 46 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap\ (c_2Emetric_2Emetric$

Let $c_2Enets_2Eetends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Eetends\ A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ ((2^{A_27b})^{A_27b}))})^{A_27a})^{(A_27a^{A_27b})}) \quad (30)$$

Definition 47 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 48 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Definition 49 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 50 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap$

Definition 51 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in ($

Definition 52 We define $c_2Emeasure_2Ecountably_additive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 53 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Esubsets A_27a \in (\quad (31)$$

$$(2^{(2^{A_27a})})^{(ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))})$$

Definition 54 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A$

Definition 55 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E3F$

Definition 56 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Espace A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))}) \quad (32)$$

Definition 57 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2$

Definition 58 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})}$

Definition 59 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2$

Definition 60 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_2$

Definition 61 We define $c_2Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A$

Definition 62 We define $c_2Eprobability_2Eprob_space$ to be $\lambda A_27a : \iota.\lambda V0p \in (ty_2Epair_2Eprod (2^{A_27$

Assume the following.

$$True \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (34)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t))))))
\end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
& A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))
\end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg (p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\
& p \ V0t))))))
\end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3))))))
\end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\
& 2. (((((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))) \Rightarrow \\
& (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27))))))
\end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\
& \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap \ (c_2Epair_2EFST \ A_27a \\
& A_27b) \ (ap \ (ap \ (c_2Epair_2E_2C \ A_27a \ A_27b) \ V0x) \ V1y)) = V0x)))
\end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\
& \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap \ (c_2Epair_2ESND \ A_27a \\
& A_27b) \ (ap \ (ap \ (c_2Epair_2E_2C \ A_27a \ A_27b) \ V0x) \ V1y)) = V1y)))
\end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& (2^{A_27a}). (\forall V2x \in A_27a. ((p \ (ap \ (ap \ (c_2Ebool_2EIN \ A_27a) \\
& V2x) \ (ap \ (ap \ (c_2Epred_set_2EINTER \ A_27a) \ V0s) \ V1t))) \Leftrightarrow ((p \ (ap \\
& (ap \ (c_2Ebool_2EIN \ A_27a) \ V2x) \ V0s)) \wedge (p \ (ap \ (ap \ (c_2Ebool_2EIN \\
& A_27a) \ V2x) \ V1t))))))
\end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0s \in (2^{A_27a}).(\forall V1t \in \\
& (2^{A_27a}).(p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ (ap\ (ap\ (c_2Epred_set_2EINTER \\
& A_27a)\ V0s)\ V1t))\ V0s)))) \wedge (\forall V2s \in (2^{A_27a}).(\forall V3t \in \\
& (2^{A_27a}).(p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ (ap\ (ap\ (c_2Epred_set_2EINTER \\
& A_27a)\ V3t)\ V2s))\ V2s))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27b}).(\forall V2x \in \\
& (ty_2Epair_2Eprod\ A_27a\ A_27b).(p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\
& A_27a\ A_27b))\ V2x)\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ A_27a\ A_27b) \\
& V0P)\ V1Q)))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ (ap\ (c_2Epair_2EFST \\
& A_27a\ A_27b)\ V2x))\ V0P)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ (ap\ (c_2Epair_2ESND \\
& A_27a\ A_27b)\ V2x))\ V1Q))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0set \in (2^{A_27a}).(\forall V1e \in \\
& (2^{A_27a}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ V1e)\ (ap\ (c_2Epred_set_2EPOW \\
& A_27a)\ V0set)))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V1e) \\
& V0set))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in (A_27b^{A_27a}).(\forall V1s \in (2^{A_27b}).(\forall V2x \in \\
& A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ (ap\ (ap\ (c_2Epred_set_2EPREIMAGE \\
& A_27a\ A_27b)\ V0f)\ V1s)))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ (ap\ V0f \\
& V2x))\ V1s))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\
& (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealx_2Ereal^{(2^{A_27a})}))). \\
& (\forall V1s \in (2^{A_27a}).(\forall V2t \in (2^{A_27a}).(((p\ (ap\ (c_2Eprobability_2Eprob_space \\
& A_27a)\ V0p)) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ V1s)\ (ap\ (c_2Eprobability_2Eevents \\
& A_27a)\ V0p)))) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ V2t)\ (ap\ (c_2Eprobability_2Eevents \\
& A_27a)\ V0p)))) \wedge (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V1s)\ V2t)))))) \Rightarrow \\
& (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ (ap\ (c_2Eprobability_2Eprob \\
& A_27a)\ V0p)\ V1s))\ (ap\ (ap\ (c_2Eprobability_2Eprob\ A_27a)\ V0p)\ V2t))))))
\end{aligned} \tag{48}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
nonempty\ A_27c & \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod \\
& (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))). (\forall V1X \in \\
& (A_27b^{A_27a}). (\forall V2Y \in (A_27c^{A_27a}). (\forall V3a \in (2^{A_27b}). \\
& (\forall V4b \in (2^{A_27c}). (((p\ (ap\ (c_2Eprobability_2Eprob_space \\
A_27a)\ V0p)) \wedge ((ap\ (c_2Eprobability_2Eevents\ A_27a)\ V0p) = (ap \\
& (c_2Epred_set_2EPOW\ A_27a)\ (ap\ (c_2Eprobability_2Eprob_space \\
A_27a)\ V0p)))) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ (ap\ (ap\ (ap\ (\\
& c_2Eprobability_2Ejoint_distribution\ A_27b\ A_27c\ A_27a)\ V0p) \\
V1X)\ V2Y)\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ A_27b\ A_27c)\ V3a)\ V4b))) \\
& (ap\ (ap\ (ap\ (c_2Eprobability_2Edistribution\ A_27c\ A_27a)\ V0p) \\
& V2Y)\ V4b)))))))))
\end{aligned}$$