thm_2Eprobability_2Ejoint__distribution__sums__1 (TMQurpnQ8i5hqoqJSBMbvbk1igYfrsVj2Fb)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2EBOUNDED *to be* $(\lambda V0v \in 2.$c_2Ebool_2ET$)$.

Definition 5 *We define* c_2Ecombin_2EC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27c}$

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A_27}$

Definition 7 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Definition 8 *We define* c_2Emarker_2EAbbrev *to be* $\lambda V0x \in 2.V0x$.

Definition 9 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 12 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0x\in A\_27a.\lambda V1y\in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b\in((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})})\tag{3}$$

Definition 13 *We define* c_2Epred__set_2EPREIMAGE *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0f\in(A\_27b^{A\_27a}).\lambda V$

Let $c\_2Emeasure\_2Esubsets:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Emeasure\_2Esubsets\ A\_27a\in((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))})\tag{4}$$

Definition 14 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a:\iota.(\lambda V0P\in(2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 15 *We define* c_2Epred__set_2EBIGUNION *to be* $\lambda A\_27a:\iota.\lambda V0P\in(2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 16 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).\lambda V1t\in(2^{A\_27a}).(ap\ ($

Let $ty\_2Enum\_2Enum:\iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum\tag{5}$$

Definition 17 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A_27a:\iota.(\lambda V0x\in A_27a.c_2Ebool_2ET)$.

Definition 18 *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0f\in(A\_27b^{A\_27a}).\lambda V1s\in(2^{A}$

Definition 19 *We define* c_2Epred__set_2Ecountable *to be* $\lambda A_27a:\iota.\lambda V0s\in(2^{A_27a}).(ap\ ($c_2Ebool_2E_3F

Definition 20 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1\in 2.(\lambda V1t2\in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t\in$

Definition 21 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).\lambda V1t\in(2^{A\_27a}).(ap\ (c.$

Let $c\_2Emeasure\_2Espace:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Emeasure\_2Espace\ A\_27a\in((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))})\tag{6}$$

Definition 22 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t\in 2.V0t))$.

Definition 23 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t\in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E $V0t)\ $c_2Ebool_2E

Definition 24 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).\lambda V1t\in(2^{A\_27a}).(ap\ (c\_2I$

Definition 25 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a:\iota.(\lambda V0x\in A_27a.c_2Ebool_2EF)$.

Definition 26 *We define* c_2Emeasure_2Esubset__class *to be* $\lambda A\_27a:\iota.\lambda V0sp\in(2^{A\_27a}).\lambda V1sts\in(2^{(2^{A\_27}}$

Definition 27 *We define* c_2Emeasure_2Ealgebra *to be* $\lambda A\_27a:\iota.\lambda V0a\in(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2}$

Definition 28 *We define* c_2Emeasure_2Esigma__algebra *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_2$

Let $c\_2Epair\_2ESND : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{7}$$

Let $c\_2Epair\_2EFST : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{8}$$

Definition 29 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27$

Definition 30 *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_$

Definition 31 *We define* c_2Epred__set_2EFUNSET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in ($

Definition 32 *We define* c_2Emeasure_2Emeasurable *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0a \in (ty\_2Epair\_2Epro$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{9}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{10}$$

Definition 33 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 34 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{11}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{12}$$

Definition 35 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

Definition 36 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 37 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{14}$$

Let $c\_2Emeasure\_2Emeasure : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Emeasure\_2Emeasure\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}) \tag{15}$$

Definition 38 *We define* c_2Eprobability_2Eprob *to be* $\lambda A\_27a : \iota.(c\_2Emeasure\_2Emeasure\ A\_27a)$.

Let $c\_2Emeasure\_2Emeasurable\_\_sets : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Emeasure\_2Emeasurable\_\_sets\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))}) \tag{16}$$

Definition 39 *We define* c_2Eprobability_2Eevents *to be* $\lambda A\_27a : \iota.(c\_2Emeasure\_2Emeasurable\_\_sets\ A\_$

Let $c\_2Emeasure\_2Em\_\_space : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Emeasure\_2Em\_\_space\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))}) \tag{17}$$

Definition 40 *We define* c_2Eprobability_2Ep__space *to be* $\lambda A\_27a : \iota.(c\_2Emeasure\_2Em\_\_space\ A\_27a)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{18}$$

Definition 41 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

Let $c\_2Ereal\_2Esum : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2En} \tag{19}$$

Definition 42 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 43 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 44 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{20}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{21}$$

Definition 45 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 (t

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{22}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{23}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Eh} \tag{24}$$

Definition 46 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

Definition 47 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Eh} \tag{25}$$

Definition 48 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 49 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{26}$$

Definition 50 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 51 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 52 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 53 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap$ (c_2Ebool_2ECONI

Let $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \tag{27}$$

Let $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \tag{28}$$

Definition 54 *We define* c_2Emetric_2Emr1 *to be* $(ap\ (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal)\ (ap\ ($c_

Let $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27}$$ (29)

Let $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \tag{30}$$

Let $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \tag{31}$$

Definition 55 *We define* c_2Emetric_2Emtop *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(a$

Let $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a^{A\_27b})}) \tag{32}$$

Definition 56 *We define* c_2Eseq_2E_2D_2D_3E *to be* $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

Definition 57 *We define* c_2Eseq_2Esums *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty$

Definition 58 *We define* c_2Epred__set_2EDISJOINT *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap$

Definition 59 *We define* c_2Emeasure_2Ecountably__additive *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod$

Definition 60 *We define* c_2Emeasure_2Epositive *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (t$

Definition 61 *We define* c_2Emeasure_2Emeasure__space *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A}$

Definition 62 *We define* c_2Eprobability_2Eprob__space *to be* $\lambda A\_27a : \iota.\lambda V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27}$

Definition 63 *We define* c_2Eprobability_2Erandom__variable *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0X \in (A\_27b^{A\_}$

Definition 64 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c$

Definition 65 *We define* c_2Eprobability_2Ejoint__distribution *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0p \in$

Definition 66 *We define* c_2Eprobability_2Edistribution *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (ty\_2Epair\_2Ep$

Definition 67 *We define* c_2Epred__set_2EPOW *to be* $\lambda A\_27a : \iota.\lambda V0set \in (2^{A\_27a}).(ap\ (c\_2Epred\_\_set\_2E$

Definition 68 *We define* c_2Epred__set_2ECROSS *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in (2$

Let $c\_2Epred\_\_set\_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EITSET\ A\_27a\ A\_27b \in (((A\_27b^{A\_27b})^{(2^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})}) \tag{33}$$

Definition 69 *We define* c_2Ereal__sigma_2EREAL__SUM__IMAGE *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2$

Definition 70 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_21 (2

Assume the following.

$$True \tag{34}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{36}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{37}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{38}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A\_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \tag{43}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P) \wedge (\forall V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V1P\ V3x))) \vee (p\ V0Q))))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{47}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \tag{48}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A))))))) \tag{49}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{50}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{51}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.(\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \land (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \land ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27)))))))) \tag{52}$$

Assume the following.

$$(\forall V0v \in 2.((p\ (ap\ \mathsf{c\_2Ebool\_2EBOUNDED}\ V0v)) \Leftrightarrow True)) \tag{53}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (2^{A\_27a}).(\forall V1y \in (2^{(2^{A\_27a})}).((ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0x)\ V1y)) = V0x))) \tag{54}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (2^{A\_27a}).(\forall V1y \in (2^{(2^{A\_27a})}).((ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0x)\ V1y)) = V1y))) \tag{55}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0sp \in (2^{A\_27a}).(p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0sp)\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EPOW}\ A\_27a)\ V0sp))))) \tag{56}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})})).(\forall V1b \in (ty\_2Epair\_2Eprod\ (2^{A\_27b})\ (2^{(2^{A\_27b})})).(\forall V2f \in (A\_27b^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (A\_27b^{A\_27a}))\ V2f)\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasurable}\ A\_27a\ A\_27b)\ V0a)\ V1b))) \Leftrightarrow ((p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}\ A\_27a)\ V0a)) \land ((p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}\ A\_27b)\ V1b)) \land ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (A\_27b^{A\_27a}))\ V2f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFUNSET}\ A\_27a\ A\_27b)\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0a))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27b)\ V1b)))) \land (\forall V3s \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27b}))\ V3s)\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27b)\ V1b))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EPREIMAGE}\ A\_27a\ A\_27b)\ V2f)\ V3s))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0a)))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0a)))))))))))) \tag{57}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\exists V1q \in A\_27a. (\exists V2r \in A\_27b.(V0x = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1q)\ V2r))))) \tag{58}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V0x))) \tag{59}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V1y))) \tag{60}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1x \in A\_27a.(\forall V2y \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1x)\ V2y)) = (ap\ (ap\ V0f\ V1x)\ V2y))))) \tag{61}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ V0s)\ V1t))\ V0s)))) \wedge (\forall V2s \in (2^{A\_27a}).(\forall V3t \in (2^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ V3t)\ V2s))\ V2s))))) \tag{62}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1y)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))) \Leftrightarrow (V0x = V1y)))) \tag{63}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0y \in A\_27b.(\forall V1s \in (2^{A\_27a}).(\forall V2f \in (A\_27b^{A\_27a}). ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V0y)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A\_27a.((V0y = (ap\ V2f\ V3x)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V1s)))))))) \tag{64}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1P \in (2^{A\_27a}).(\forall V2Q \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (A\_27b^{A\_27a}))\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFUNSET}\ A\_27a\ A\_27b)\ V1P)\ V2Q))) \Leftrightarrow (\forall V3x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ (ap\ V0f\ V3x))\ V2Q)))))))) \tag{65}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \Rightarrow (\forall V1f \in (A\_27b^{A\_27a}).(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V1f)\ V0s)))))) \tag{66}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ECROSS}\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27b)\ V1y)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27b))) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y))\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)))))) \tag{67}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0set \in (2^{A\_27a}).(\forall V1e \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ V1e)\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EPOW}\ A\_27a)\ V0set))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V1e)\ V0set))))) \tag{68}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (2^{A\_27a}).(\forall V1b \in (2^{(2^{A\_27a})}).(\forall V2c \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}).((ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))\ V0a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))\ V1b)\ V2c))) = V0a)))) \tag{69}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (2^{A\_27a}).(\forall V1b \in (2^{(2^{A\_27a})}).(\forall V2c \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}). ((ap\ (\mathsf{c\_2Eprobability\_2Eevents}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))\ V0a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))\ V1b)\ V2c))) = V1b)))) \tag{70}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (2^{A\_27a}).(\forall V1b \in (2^{(2^{A\_27a})}).(\forall V2c \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}). ((ap\ (\mathsf{c\_2Eprobability\_2Eprob}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))\ V0a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))\ V1b)\ V2c))) = V2c)))) \tag{71}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))). (((p\ (ap\ (\mathsf{c\_2Eprobability\_2Eprob\_\_space}\ A\_27a)\ V0p)) \wedge ((\forall V1x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))\ (ap\ (\mathsf{c\_2Eprobability\_2Eevents}\ A\_27a)\ V0p))))) \wedge (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p))))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ (\lambda V2x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Eprobability\_2Eprob}\ A\_27a)\ V0p)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V2x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))))\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p)) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))) \tag{72}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))).(\forall V1X \in (A\_27b^{A\_27a}).(\forall V2s \in (ty\_2Epair\_2Eprod\ (2^{A\_27b})\ (2^{(2^{A\_27b})})). ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eprobability\_2Erandom\_\_variable}\ A\_27a\ A\_27b)\ V1X)\ V0p)\ V2s)) \Rightarrow (p\ (ap\ (\mathsf{c\_2Eprobability\_2Eprob\_\_space}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27b})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27b})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27b})}))))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27b)\ V2s))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{A\_27b})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27b})}))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27b)\ V2s))\ (ap\ (ap\ (\mathsf{c\_2Eprobability\_2Edistribution}\ A\_27b\ A\_27a)\ V0p)\ V1X))))))))) \tag{73}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))).(\forall V1X \in (A\_27b^{A\_27a}).(\forall V2Y \in (A\_27c^{A\_27a}).(\forall V3a \in (2^{A\_27b}). (((p\ (ap\ (\mathsf{c\_2Eprobability\_2Eprob\_\_space}\ A\_27a)\ V0p)) \land ((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p))) \land ((ap\ (\mathsf{c\_2Eprobability\_2Eevents}\ A\_27a)\ V0p) = (ap\ (\mathsf{c\_2Epred\_\_set\_2EPOW}\ A\_27a)\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p))))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Eprobability\_2Edistribution}\ A\_27b\ A\_27a)\ V0p)\ V1X)\ V3a) = (ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ A\_27c)\ (\lambda V4x \in A\_27c.(ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eprobability\_2Ejoint\_\_distribution}\ A\_27b\ A\_27c\ A\_27a)\ V0p)\ V1X)\ V2Y)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ECROSS}\ A\_27b\ A\_27c)\ V3a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27c)\ V4x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27c))))))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27c)\ V2Y)\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p))))))))) \tag{74}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0P)) \Rightarrow (\forall V1f \in (ty\_2Erealax\_2Ereal^{A\_27a}). ((ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ V1f)\ V0P) = (ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ (\lambda V2x \in A\_27a.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0P))\ (ap\ V1f\ V2x))\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))))\ V0P))))) \tag{75}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1s\_27 \in (2^{A\_27b}).(\forall V2f \in ((ty\_2Erealax\_2Ereal^{A\_27b})^{A\_27a}).(((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \wedge (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27b)\ V1s\_27))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ (\lambda V3x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ A\_27b)\ (ap\ V2f\ V3x))\ V1s\_27)))\ V0s) = (ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (\lambda V4x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(ap\ (ap\ V2f\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ V4x))\ (ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ V4x))))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ECROSS}\ A\_27a\ A\_27b)\ V0s)\ V1s\_27))))))) \tag{76}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{77}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{78}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{79}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{80}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{81}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \tag{82}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{83}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{84}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{85}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{86}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{87}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{88}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{89}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{90}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{91}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))).(\forall V1X \in (A\_27b^{A\_27a}).(\forall V2Y \in (A\_27c^{A\_27a}).(((p\ (ap\ (\mathsf{c\_2Eprobability\_2Eprob\_\_space}\ A\_27a)\ V0p)) \wedge ((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p))) \wedge ((ap\ (\mathsf{c\_2Eprobability\_2Eevents}\ A\_27a)\ V0p) = (ap\ (\mathsf{c\_2Epred\_\_set\_2EPOW}\ A\_27a)\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p))))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27c))\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27b\ A\_27c\ ty\_2Erealax\_2Ereal)\ (\lambda V3x \in A\_27b.(\lambda V4y \in A\_27c.(ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eprobability\_2Ejoint\_\_distribution}\ A\_27b\ A\_27c\ A\_27a)\ V0p)\ V1X)\ V2Y)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27c))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ A\_27c)\ V3x)\ V4y))\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27c)))))))))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ECROSS}\ A\_27b\ A\_27c)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V1X)\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p)))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27c)\ V2Y)\ (ap\ (\mathsf{c\_2Eprobability\_2Ep\_\_space}\ A\_27a)\ V0p)))) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))))$$