

thm_2Eprobability_2Emarginal__joint__zero (TMG6yTq7C5UB6ikAWejrixkhiT82TriUn7W)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ecombin_2E_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a})).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (1)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (2)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (3)$$

Definition 6 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a})).(ap\ V1f\ V0x)))$

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (4)$$

Definition 9 We define c_2Epair_2E2C to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (5)$$

Definition 10 We define $c_2Epred_set_2ECROSS$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0P \in (2^{A_27a}). \lambda V1Q \in (2^{A_27b}).$

Definition 11 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ ($

Definition 12 We define $c_2Epred_set_2EPOW$ to be $\lambda A_27a : \iota. \lambda V0set \in (2^{A_27a}). (ap\ (c_2Epred_set_2E$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (6)$$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in \\ ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \end{aligned} \quad (7)$$

Definition 13 We define $c_2Eprobability_2Ep_space$ to be $\lambda A_27a : \iota. (c_2Emeasure_2Em_space\ A_27a).$

Definition 14 We define $c_2Epred_set_2EPREIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V$

Definition 15 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasure\ A_27a \in (\\ (ty_2Erealax_2Ereal^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \end{aligned} \quad (8)$$

Definition 16 We define $c_2Eprobability_2Eprob$ to be $\lambda A_27a : \iota. (c_2Emeasure_2Emeasure\ A_27a).$

Definition 17 We define $c_2Eprobability_2Edistribution$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0p \in (ty_2Epair_2E$

Definition 18 We define $c_2Eprobability_2Ejoint_distribution$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0p \in$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{9}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{10}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{11}$$

Definition 19 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \tag{12}$$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets \\ & A_27a \in ((2^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))})) \end{aligned} \tag{13}$$

Definition 20 We define $c_2Eprobability_2Eevents$ to be $\lambda A_27a : \iota. (c_2Emeasure_2Emeasurable_sets\ A_27a)$.

Definition 21 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{14}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{15}$$

Definition 22 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ m)$.

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{16}$$

Definition 23 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic_2E_2B\ n))$.

Definition 24 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 25 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2ET)$.

Definition 26 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in$

Definition 27 We define c_Emin_E40 to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A) \text{ of type } \iota \Rightarrow \iota.$

Definition 28 We define c_Ebool_E3F to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap \ V0P \ (ap \ (c_Emin_E40 \ A_27a) \ V0P)))$

Definition 29 We define $c_Epred_set_EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap \ (c_Epred_set_E1 \ A_27a) \ V0P)$

Let $c_Ereal_Esum : \iota$ be given. Assume the following.

$$c_Ereal_Esum \in ((ty_Erealax_Ereal^{(ty_Erealax_Ereal^{ty_Eenum_Eenum})})^{(ty_Epair_Eprod \ ty_Eenum_Eenum)})^{(ty_Epair_Eprod \ ty_Eenum_Eenum)} \quad (17)$$

Definition 30 We define c_Ebool_E2F to be $(ap \ (c_Ebool_E21 \ 2) \ (\lambda V0t \in 2. V0t))$.

Definition 31 We define c_Ebool_E7E to be $(\lambda V0t \in 2. (ap \ (ap \ c_Emin_E3D_3D_3E \ V0t) \ c_Ebool_E2F \ V0t))$

Definition 32 We define $c_Eprim_rec_E3C$ to be $\lambda V0m \in ty_Eenum_Eenum. \lambda V1n \in ty_Eenum_Eenum. \lambda V2t \in 2. (ap \ (c_Eprim_rec_E3C \ V0m \ V1n) \ V2t)$

Definition 33 We define $c_Earithmic_E3E$ to be $\lambda V0m \in ty_Eenum_Eenum. \lambda V1n \in ty_Eenum_Eenum. \lambda V2t \in 2. (ap \ (c_Earithmic_E3E \ V0m \ V1n) \ V2t)$

Definition 34 We define $c_Ebool_E5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap \ (c_Ebool_E21 \ 2) \ (\lambda V2t \in 2. (ap \ (c_Ebool_E5C_2F \ V0t1 \ V1t2) \ V2t)) \ V0t1)))$

Definition 35 We define $c_Earithmic_E3E_3D$ to be $\lambda V0m \in ty_Eenum_Eenum. \lambda V1n \in ty_Eenum_Eenum. \lambda V2t \in 2. (ap \ (c_Earithmic_E3E_3D \ V0m \ V1n) \ V2t)$

Let $ty_Ehreal_Ehreal : \iota$ be given. Assume the following.

$$nonempty \ ty_Ehreal_Ehreal \quad (18)$$

Let $c_Erealax_Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_REP_CLASS \in ((2^{(ty_Epair_Eprod \ ty_Ehreal_Ehreal \ ty_Ehreal_Ehreal)})^{(ty_Epair_Eprod \ ty_Ehreal_Ehreal)})^{(ty_Epair_Eprod \ ty_Ehreal_Ehreal)} \quad (19)$$

Definition 36 We define $c_Erealax_Ereal_REP$ to be $\lambda V0a \in ty_Erealax_Ereal. (ap \ (c_Emin_E40 \ A_27a) \ V0a)$

Let $c_Erealax_Etrealm_neg : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_neg \in ((ty_Epair_Eprod \ ty_Ehreal_Ehreal \ ty_Ehreal_Ehreal)^{ty_Epair_Eprod \ ty_Ehreal_Ehreal})^{(ty_Epair_Eprod \ ty_Ehreal_Ehreal)} \quad (20)$$

Let $c_Erealax_Etrealm_eq : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_eq \in ((2^{(ty_Epair_Eprod \ ty_Ehreal_Ehreal \ ty_Ehreal_Ehreal)})^{(ty_Epair_Eprod \ ty_Ehreal_Ehreal)})^{(ty_Epair_Eprod \ ty_Ehreal_Ehreal)} \quad (21)$$

Let $c_Erealax_Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_ABS_CLASS \in (ty_Erealax_Ereal^{(2^{(ty_Epair_Eprod \ ty_Ehreal_Ehreal \ ty_Ehreal_Ehreal)})})^{(ty_Epair_Eprod \ ty_Ehreal_Ehreal)} \quad (22)$$

Definition 37 We define $c_Erealax_Ereal_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod \ ty_Ehreal_Ehreal \ ty_Ehreal_Ehreal) \rightarrow ty_Erealax_Ereal$

Definition 47 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap$
 Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Etends \\ & A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ (2^{A_27b})^{A_27b})}))_{A_27a})(A_27a^{A_27b}) \end{aligned} \quad (30)$$

Definition 48 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 49 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Definition 50 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 51 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap$

Definition 52 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A_27b}).$

Definition 53 We define $c_2Emeasure_2Ecountably_additive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 54 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2E$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in (\\ & (2^{(2^{A_27a})})(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))) \end{aligned} \quad (31)$$

Definition 55 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a}).$

Definition 56 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_3F$

Definition 57 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2E$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in (((2^{A_27a})(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})})))) \end{aligned} \quad (32)$$

Definition 58 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2E$

Definition 59 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})}).$

Definition 60 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}).$

Definition 61 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}).$

Definition 62 We define $c_2Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2E$

Definition 63 We define $c_2Eprobability_2Eprob_space$ to be $\lambda A_27a : \iota.\lambda V0p \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2E$

Assume the following.

$$True \quad (33)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (37)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((\forall V3x \in A_27a.(p \ (ap \ V0P \ V3x))) \wedge (\forall V4x \in A_27a.(p \ (ap \ V1Q \ V4x))))))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p \ V1B) \wedge (p \ V2C)) \vee (p \ V0A)) \Leftrightarrow (((p \ V1B) \vee (p \ V0A)) \wedge ((p \ V2C) \vee (p \ V0A))))))) \quad (43)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (44)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.((((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))))) \Rightarrow (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27))))) \quad (45)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27b.((ap \ (c_2Epair_2EFST \ A_27a \ A_27b) \ (ap \ (ap \ (c_2Epair_2E_2C \ A_27a \ A_27b) \ V0x) \ V1y)) = V0x))) \quad (46)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27b.((ap \ (c_2Epair_2ESND \ A_27a \ A_27b) \ (ap \ (ap \ (c_2Epair_2E_2C \ A_27a \ A_27b) \ V0x) \ V1y)) = V1y))) \quad (47)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in (2^{A_27a}).(\forall V2x \in A_27a.((p \ (ap \ (ap \ (c_2Ebool_2EIN \ A_27a) \ V2x) \ (ap \ (ap \ (c_2Epred_set_2EINTER \ A_27a) \ V0s) \ V1t))) \Leftrightarrow ((p \ (ap \ (ap \ (c_2Ebool_2EIN \ A_27a) \ V2x) \ V0s)) \wedge (p \ (ap \ (ap \ (c_2Ebool_2EIN \ A_27a) \ V2x) \ V1t))))))) \quad (48)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow ((\forall V0s \in (2^{A_27a}).(\forall V1t \in (2^{A_27a}).(p \ (ap \ (ap \ (c_2Epred_set_2ESUBSET \ A_27a) \ (ap \ (ap \ (c_2Epred_set_2EINTER \ A_27a) \ V0s) \ V1t)) \ V0s)))) \wedge (\forall V2s \in (2^{A_27a}).(\forall V3t \in (2^{A_27a}).(p \ (ap \ (ap \ (c_2Epred_set_2ESUBSET \ A_27a) \ (ap \ (ap \ (c_2Epred_set_2EINTER \ A_27a) \ V3t) \ V2s)) \ V2s)))))) \quad (49)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27b}).(\forall V2x \in (ty_2Epair_2Eprod \ A_27a \ A_27b).((p \ (ap \ (ap \ (c_2Ebool_2EIN \ (ty_2Epair_2Eprod \ A_27a \ A_27b)) \ V2x) \ (ap \ (ap \ (c_2Epred_set_2ECROSS \ A_27a \ A_27b) \ V0P) \ V1Q))) \Leftrightarrow ((p \ (ap \ (ap \ (c_2Ebool_2EIN \ A_27a) \ (ap \ (c_2Epair_2EFST \ A_27a \ A_27b) \ V2x)) \ V0P)) \wedge (p \ (ap \ (ap \ (c_2Ebool_2EIN \ A_27b) \ (ap \ (c_2Epair_2ESND \ A_27a \ A_27b) \ V2x)) \ V1Q)))))) \quad (50)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0set \in (2^{A.27a}). (\forall V1e \in \\ (2^{A.27a}). ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (2^{A.27a})\ V1e)\ (ap\ (c.2Epred_set.2EPOW \\ A.27a)\ V0set))) \Leftrightarrow (p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET\ A.27a)\ V1e) \\ V0set)))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0f \in (A.27b^{A.27a}). (\forall V1s \in (2^{A.27b}). (\forall V2x \in \\ A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ (ap\ (ap\ (c.2Epred_set.2EPREIMAGE \\ A.27a\ A.27b)\ V0f)\ V1s))) \Leftrightarrow (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27b)\ (ap\ V0f \\ V2x))\ V1s)))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0p \in (ty.2Epair.2Eprod \\ (2^{A.27a})\ (ty.2Epair.2Eprod\ (2^{(2^{A.27a})})\ (ty.2Erealx.2Ereal^{(2^{A.27a})}))). \\ (\forall V1s \in (2^{A.27a}). (((p\ (ap\ (c.2Eprobability.2Eprob_space \\ A.27a)\ V0p)) \wedge (p\ (ap\ (ap\ (c.2Ebool.2EIN\ (2^{A.27a})\ V1s)\ (ap\ (c.2Eprobability.2Eevents \\ A.27a)\ V0p)))) \Rightarrow (p\ (ap\ (ap\ c.2Ereal.2Ereal_lte\ (ap\ c.2Ereal.2Ereal_of_num \\ c.2Enum.2E0))\ (ap\ (ap\ (c.2Eprobability.2Eprob\ A.27a)\ V0p)\ V1s)))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0p \in (ty.2Epair.2Eprod \\ (2^{A.27a})\ (ty.2Epair.2Eprod\ (2^{(2^{A.27a})})\ (ty.2Erealx.2Ereal^{(2^{A.27a})}))). \\ (\forall V1s \in (2^{A.27a}). (\forall V2t \in (2^{A.27a}). (((p\ (ap\ (c.2Eprobability.2Eprob_space \\ A.27a)\ V0p)) \wedge ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (2^{A.27a})\ V1s)\ (ap\ (c.2Eprobability.2Eevents \\ A.27a)\ V0p)))) \wedge ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (2^{A.27a})\ V2t)\ (ap\ (c.2Eprobability.2Eevents \\ A.27a)\ V0p)))) \wedge (p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET\ A.27a)\ V1s)\ V2t)))))) \Rightarrow \\ (p\ (ap\ (ap\ c.2Ereal.2Ereal_lte\ (ap\ (ap\ (c.2Eprobability.2Eprob \\ A.27a)\ V0p)\ V1s))\ (ap\ (ap\ (c.2Eprobability.2Eprob\ A.27a)\ V0p)\ V2t)))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty.2Erealx.2Ereal. (\forall V1y \in ty.2Erealx.2Ereal. \\ (((p\ (ap\ (ap\ c.2Ereal.2Ereal_lte\ V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c.2Ereal.2Ereal_lte \\ V1y)\ V0x)))) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (55)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (56)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (57)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow ((p \ V0A) \Rightarrow False) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \quad (58)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p \ V0A)) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow ((p \ V0A) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \quad (59)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \ V0A)) \Rightarrow False) \Rightarrow (((p \ V0A) \Rightarrow False) \Rightarrow False))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \Leftrightarrow (p \ V2r)) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow ((p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p)))))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow ((p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow ((p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((\neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p)))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p))))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q))))) \quad (69)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \quad (70)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow \forall A_27c. \\ & nonempty \ A_27c \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \ (2^{A_27a}) \ (ty_2Epair_2Eprod \\ & \ (2^{(2^{A_27a})}) \ (ty_2Erealax_2Ereal^{(2^{A_27a})}))).(\forall V1X \in \\ & (A_27b^{A_27a}).(\forall V2Y \in (A_27c^{A_27a}).(\forall V3s \in (2^{A_27b}). \\ & (\forall V4t \in (2^{A_27c}).(((p \ (ap \ (c_2Eprobability_2Eprob_space \\ & A_27a) \ V0p))) \wedge (((ap \ (c_2Eprobability_2Eevents \ A_27a) \ V0p) = (ap \\ & (c_2Epred_set_2EPOW \ A_27a) \ (ap \ (c_2Eprobability_2Eprob_space \\ & A_27a) \ V0p))) \wedge (((ap \ (ap \ (ap \ (c_2Eprobability_2Edistribution \\ & A_27b \ A_27a) \ V0p) \ V1X) \ V3s) = (ap \ c_2Ereal_2Ereal_of_num \ c_2Enum_2E0)) \vee \\ & ((ap \ (ap \ (ap \ (c_2Eprobability_2Edistribution \ A_27c \ A_27a) \ V0p) \\ & V2Y) \ V4t) = (ap \ c_2Ereal_2Ereal_of_num \ c_2Enum_2E0)))))) \Rightarrow ((\\ & ap \ (ap \ (ap \ (ap \ (c_2Eprobability_2Ejoint_distribution \ A_27b \ A_27c \\ & A_27a) \ V0p) \ V1X) \ V2Y) \ (ap \ (ap \ (c_2Epred_set_2ECROSS \ A_27b \ A_27c) \\ & V3s) \ V4t)) = (ap \ c_2Ereal_2Ereal_of_num \ c_2Enum_2E0)))))) \end{aligned}$$