

thm_2Eproduct_2ENPRODUCT__PAIR (TMQnN2SxB5LziiMQH8w6aFTAMrGftnkFVGp)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 8 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 10 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 11 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 12 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 13 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 14 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 15 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 16 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 17 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2$

Definition 18 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (7)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 19 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Emin$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \quad (9)$$

Definition 20 We define $c_2Eiterate_2E_2E_2E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 21 We define $c_2Eiterate_2Eneutral$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap\ (c_2Emin$

Definition 22 We define $c_2Eiterate_2Emonoidal$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap\ (ap\ c_2Emin$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (10)$$

Definition 23 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 24 We define $c_2Eiterate_2Esupport$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0op \in ((A_27b^{A_27b})^{A_27b}). \lambda V$

Definition 25 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (c_2Ebool_2ECOND\ V0t\ V1t1\ V2t2)))$

Definition 26 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Epred_set_2EINSERT\ V0x\ V1s))$

Definition 27 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2ECOND\ V0x)$.

Definition 28 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap\ (c_2Ebool_2ECOND\ V0s))$

Definition 29 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in ((A_27a^{A_27a})^{A_27b}). \lambda V$

Definition 30 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0op \in ((A_27b^{A_27b})^{A_27b}). \lambda V$

Definition 31 We define $c_2Eproduct_2Enproduct$ to be $\lambda A_27a : \iota. (ap\ (c_2Eiterate_2Eiterate\ A_27a\ ty_2Enum_2Enum))$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0op \in ((A_27a^{A_27a})^{A_27a}). \\ & ((p\ (ap\ (c_2Eiterate_2Emonoidal\ A_27a)\ V0op)) \Rightarrow (\forall V1f \in (\\ & A_27a^{ty_2Enum_2Enum}). (\forall V2m \in ty_2Enum_2Enum. (\forall V3n \in \\ & ty_2Enum_2Enum. ((ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ ty_2Enum_2Enum \\ & A_27a)\ V0op)\ (ap\ (ap\ c_2Eiterate_2E_2E_2E\ (ap\ (ap\ c_2Earithmetic_2E_2A \\ & (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))) \\ & V2m))\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (ap\ c_2Earithmetic_2E_2A \\ & (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))) \\ & V3n))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO))))))\ V1f) = (ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate \\ & ty_2Enum_2Enum\ A_27a)\ V0op)\ (ap\ (ap\ c_2Eiterate_2E_2E_2E\ V2m) \\ & V3n))\ (\lambda V4i \in ty_2Enum_2Enum. (ap\ (ap\ V0op\ (ap\ V1f\ (ap\ (ap\ c_2Earithmetic_2E_2A \\ & (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))) \\ & V4i)))\ (ap\ V1f\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (ap\ c_2Earithmetic_2E_2A \\ & (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))) \\ & V4i))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)))))))))) \end{aligned} \quad (13)$$

Assume the following.

$$(p (ap (c_Eiterate_Emonoidal ty_Eenum_Eenum) c_Earithmic_E_E2A)) \quad (14)$$

Theorem 1

$$\begin{aligned} & (\forall V0f \in (ty_Eenum_Eenum^{ty_Eenum_Eenum}). (\forall V1m \in \\ & ty_Eenum_Eenum. (\forall V2n \in ty_Eenum_Eenum. ((ap (ap (c_Eproduct_E_Eproduct \\ & ty_Eenum_Eenum) (ap (ap c_Eiterate_E_E2E (ap (ap c_Earithmic_E_E2A \\ & (ap c_Earithmic_E_E2ENUMERAL (ap c_Earithmic_E_E2EBIT2 c_Earithmic_E_E2EZERO))) \\ & V1m)) (ap (ap c_Earithmic_E_E2E_E2B (ap (ap c_Earithmic_E_E2E_E2A \\ & (ap c_Earithmic_E_E2ENUMERAL (ap c_Earithmic_E_E2EBIT2 c_Earithmic_E_E2EZERO))) \\ & V2n)) (ap c_Earithmic_E_E2ENUMERAL (ap c_Earithmic_E_E2EBIT1 \\ & c_Earithmic_E_E2EZERO)))))) V0f) = (ap (ap (c_Eproduct_E_Eproduct \\ & ty_Eenum_Eenum) (ap (ap c_Eiterate_E_E2E_E2E V1m) V2n)) (\lambda V3i \in \\ & ty_Eenum_Eenum. (ap (ap c_Earithmic_E_E2E_E2A (ap V0f (ap (ap c_Earithmic_E_E2E_E2A \\ & (ap c_Earithmic_E_E2ENUMERAL (ap c_Earithmic_E_E2EBIT2 c_Earithmic_E_E2EZERO))) \\ & V3i))) (ap V0f (ap (ap c_Earithmic_E_E2E_E2B (ap (ap c_Earithmic_E_E2E_E2A \\ & (ap c_Earithmic_E_E2ENUMERAL (ap c_Earithmic_E_E2EBIT2 c_Earithmic_E_E2EZERO))) \\ & V3i)) (ap c_Earithmic_E_E2ENUMERAL (ap c_Earithmic_E_E2EBIT1 \\ & c_Earithmic_E_E2EZERO))))))))))))) \end{aligned}$$