

thm_2Eproduct_2ENPRODUCT__POS__LT__NUMSEG (TMGGGqtsc2dQxLxUR2fHzqu52M5hEiQ1Wdf)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Definition 9 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

Definition 10 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40$

Definition 11 We define `c_2Eprim_rec_2E_3C` to be $\lambda V0m \in \text{ty_2Enum_2Enum}. \lambda V1n \in \text{ty_2Enum_2Enum}$

Definition 12 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap (c_2Ebool_2E_21 } 2) (\lambda V2t \in$

Definition 13 We define `c_2Earithmetic_2E_3C_3D` to be $\lambda V0m \in \text{ty_2Enum_2Enum}. \lambda V1n \in \text{ty_2Enum_2}$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty (ty_2Epair_2Eprod } A0 \ A1) \quad (5)$$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Epair_2EABS_prod } A. 27a \ A. 27b \in ((\text{ty_2Epair_2Eprod } A. 27a \ A. 27b))^{(2^{A-27b})^{A-27a}} \quad (6)$$

Definition 14 We define `c_2Epair_2E_2C` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0x \in A. 27a. \lambda V1y \in A. 27b. (\text{ap (c_2}$

Let `c_2Epred_set_2EGSPEC` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Epred_set_2EGSPEC } A. 27a \ A. 27b \in ((2^{A-27a})^{(\text{ty_2Epair_2Eprod } A. 27a \ 2)^{A-27b}}) \quad (7)$$

Definition 15 We define `c_2Eiterate_2E_2E_2E` to be $\lambda V0m \in \text{ty_2Enum_2Enum}. \lambda V1n \in \text{ty_2Enum_2Enum}$

Let `c_2Earithmetic_2E_2A` : ι be given. Assume the following.

$$\text{c_2Earithmetic_2E_2A} \in ((\text{ty_2Enum_2Enum}^{\text{ty_2Enum_2Enum}})^{\text{ty_2Enum_2Enum}}) \quad (8)$$

Definition 16 We define `c_2Eiterate_2Eneutral` to be $\lambda A. 27a : \iota. \lambda V0op \in ((A. 27a^{A-27a})^{A-27a}). (\text{ap (c_2Emin$

Definition 17 We define `c_2Ebool_2EIN` to be $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f \ V0x))$

Definition 18 We define `c_2Eiterate_2Esupport` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0op \in ((A. 27b^{A-27b})^{A-27b}). \lambda V$

Definition 19 We define `c_2Ebool_2ECOND` to be $\lambda A. 27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. 27a. (\lambda V2t2 \in A. 27a. ($

Definition 20 We define `c_2Epred_set_2EINSERT` to be $\lambda A. 27a : \iota. \lambda V0x \in A. 27a. \lambda V1s \in (2^{A-27a}). (\text{ap (c_2}$

Definition 21 We define `c_2Epred_set_2EEMPTY` to be $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. \text{c_2Ebool_2EF}).$

Definition 22 We define `c_2Epred_set_2EFINITE` to be $\lambda A. 27a : \iota. \lambda V0s \in (2^{A-27a}). (\text{ap (c_2Ebool_2E_21 } 2)$

Definition 23 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27b}).\lambda V$.

Definition 24 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27a}).\lambda V$.

Definition 25 We define $c_2Eproduct_2Enproduct$ to be $\lambda A_27a : \iota.(ap (c_2Eiterate_2Eiterate A_27a ty_2En$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{9}$$

Definition 26 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Assume the following.

$$True \tag{10}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t))) \tag{11}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow \neg (p V0t)))))) \end{aligned} \tag{13}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))) \tag{14}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg (p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p V0t)))))) \end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ & 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& p (ap (c_2Epred_set_2EFINITE ty_2Enum_2Enum) (ap (ap c_2Eiterate_2E_2E_2E \\
& V0m) V1n))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) \\
& V2p) (ap (ap c_2Eiterate_2E_2E_2E V0m) V1n))) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V2p)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V2p) V1n))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0f \in (ty_2Enum_2Enum^{A_27a}). \\
& (\forall V1s \in (2^{A_27a}). (((p (ap (c_2Epred_set_2EFINITE A_27a) \\
& V1s)) \wedge (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) \\
& V1s)) \Rightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap V0f V2x)))))) \Rightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap (ap (c_2Eproduct_2Enproduct \\
& A_27a) V1s) V0f))))))
\end{aligned} \tag{20}$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}). (\forall V1m \in \\
& ty_2Enum_2Enum. (\forall V2n \in ty_2Enum_2Enum. ((\forall V3x \in \\
& ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D V1m) V3x)) \wedge \\
& (p (ap (ap c_2Earithmetic_2E_3C_3D V3x) V2n))) \Rightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& c_2Enum_2E0) (ap V0f V3x)))))) \Rightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) \\
& (ap (ap (c_2Eproduct_2Enproduct ty_2Enum_2Enum) (ap (ap c_2Eiterate_2E_2E_2E \\
& V1m) V2n)) V0f))))))
\end{aligned}$$