

# thm\_2Eproduct\_2EPRODUCT\_LE (TMU3sSdP3y6djGkuPyabrsZRazctrDcG2wc)

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**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define `c_2Ebool_2ET` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 4** We define `c_2Ebool_2E_3F` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \ (\text{ap } (\text{c\_2Emin\_2E\_40 } A \ 27a))))$

**Definition 5** We define `c_2Ecombin_2EK` to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. (\lambda V0x \in A. 27a. (\lambda V1y \in A. 27b. V0x))$

**Definition 6** We define `c_2Ecombin_2ES` to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. (\lambda V0f \in ((A. 27c^{A-27b})^{A-27a}))$

**Definition 7** We define `c_2Ecombin_2EI` to be  $\lambda A. 27a : \iota. (\text{ap } (\text{ap } (\text{c\_2Ecombin\_2ES } A. 27a \ (A. 27a^{A-27a}) \ A. 27a)))$

**Definition 8** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A-27a}))))$

**Definition 9** We define `c_2Ebool_2EF` to be  $(\text{ap } (\text{c\_2Ebool\_2E\_21 } 2)) (\lambda V0t \in 2. V0t)$ .

**Definition 10** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow p \ Q)$  of type  $\iota$ .

**Definition 11** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2)) (\lambda V2t \in 2. V2t)))$

**Definition 12** We define `c_2Ebool_2ECOND` to be  $\lambda A. 27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. 27a. (\lambda V2t2 \in A. 27a. (V1t1 \wedge V2t2))))$

**Definition 13** We define `c_2Ebool_2EIN` to be  $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f \ V0x)))$

**Definition 14** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2)) (\lambda V2t \in 2. V2t)))$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty\_2Epair\_2Eprod } A0 \ A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (2)$$

**Definition 15** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap\ (c\_2E$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (3)$$

**Definition 16** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A\_27a}). (ap\ (c\_2E$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (4)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (5)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (6)$$

**Definition 17** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 18** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (8)$$

**Definition 19** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

**Definition 20** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic$

**Definition 21** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

**Definition 22** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2EF)$ .

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (10)$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (11)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})_{ty\_2Erealax}) \quad (12)$$

**Definition 23** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ t$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (13)$$

Let  $c\_2Erealax\_2Etrealeq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealeq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (14)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (15)$$

**Definition 24** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 25** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

**Definition 26** We define  $c\_2Eiterate\_2Eneutral$  to be  $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (c\_2Emin$

**Definition 27** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E$

**Definition 28** We define  $c\_2Eiterate\_2Esupport$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

**Definition 29** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E\_21\ 2$

**Definition 30** We define  $c\_2Eiterate\_2EITSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}).\lambda V$

**Definition 31** We define  $c\_2Eiterate\_2Eiterate$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

**Definition 32** We define  $c\_2Eproduct\_2Eproduct$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Eiterate\_2Eiterate\ A\_27a\ ty\_2Ereal$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (16)$$

Let  $c\_2Erealax\_2Etreallt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreallt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (17)$$

**Definition 33** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 34** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (21)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (22)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (26)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow \\
& True))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\
& A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0t1 \in A.27a.(\forall V1t2 \in \\
& A.27a.(((ap \ (ap \ (ap \ (c.2Ebool.2ECOND \ A.27a) \ c.2Ebool.2ET) \ V0t1) \\
& V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c.2Ebool.2ECOND \ A.27a) \ c.2Ebool.2EF) \\
& V0t1) \ V1t2) = V1t2))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in \\
& A.27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p \ (ap \ V0P \ V2x))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\
& 2.(((\forall V2x \in A.27a.(p \ (ap \ V0P \ V2x))) \wedge (p \ V1Q)) \Leftrightarrow (\forall V3x \in \\
& A.27a.((p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & 2.(((\exists V2x \in A.27a.(p \ (ap \ V0P \ V2x))) \vee (p \ V1Q))) \Leftrightarrow (\exists V3x \in \\ & A.27a.((p \ (ap \ V0P \ V3x)) \vee (p \ V1Q)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ & 2^{A.27a}).(((p \ V0P) \vee (\exists V2x \in A.27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\exists V3x \in \\ & A.27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & 2.((\exists V2x \in A.27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ V1Q))) \Leftrightarrow ((\exists V3x \in \\ & A.27a.(p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ & 2^{A.27a}).(((\exists V2x \in A.27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\ & V0P) \wedge (\exists V3x \in A.27a.(p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee ( \\ & (p \ V1B) \vee (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee \\ & (p \ V0A)))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg( \\ & p \ V0A) \vee \neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A) \wedge \neg(p \ V1B)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee ( \\ & (p \ V1B) \wedge (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \wedge ((p \ V0A) \vee (p \ V2C)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V1B) \wedge \\ & (p \ V2C) \vee (p \ V0A)) \Leftrightarrow (((p \ V1B) \vee (p \ V0A)) \wedge ((p \ V2C) \vee (p \ V0A)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\ & ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \end{aligned} \quad (45)$$

Assume the following.

$$2.(((p \ V0x) \Leftrightarrow (p \ V1x_{.27})) \wedge ((p \ V1x_{.27}) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_{.27})))) \Rightarrow \\ (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_{.27}) \Rightarrow (p \ V3y_{.27})))) \quad (46)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ (\forall V2x \in A_{.27a}. (\forall V3x_{.27} \in A_{.27a}. (\forall V4y \in A_{.27a}. \\ (\forall V5y_{.27} \in A_{.27a}. (((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge ((p \ V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y_{.27})))) \Rightarrow ((ap \ (ap \ (ap \ (c_{.2Ebool\_2ECOND} \ A_{.27a}) \\ V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c_{.2Ebool\_2ECOND} \ A_{.27a}) \ V1Q) \ V3x_{.27}) \\ V5y_{.27})))))))))) \quad (47)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty \ A_{.27b} \Rightarrow ( \\ \forall V0P \in (2^{A_{.27b}})^{A_{.27a}}. ((\forall V1x \in A_{.27a}. (\exists V2y \in \\ A_{.27b}. (p \ (ap \ (ap \ V0P \ V1x) \ V2y)))) \Leftrightarrow (\exists V3f \in (A_{.27b})^{A_{.27a}}. ( \\ \forall V4x \in A_{.27a}. (p \ (ap \ (ap \ V0P \ V4x) \ (ap \ V3f \ V4x))))))) \quad (48)$$

Assume the following.

$$(\forall V0r \in 2. (\forall V1p \in 2. (\forall V2q \in 2. (((p \ V1p) \wedge \\ (p \ V2q) \Rightarrow (p \ V0r)) \Leftrightarrow ((p \ V1p) \Rightarrow ((p \ V2q) \Rightarrow (p \ V0r)))))) \quad (49)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. ((ap \ (c_{.2Ecombin\_2EI} \\ A_{.27a}) \ V0x) = V0x)) \quad (50)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. (\neg(p \ (ap \ (ap \\ (c_{.2Ebool\_2EIN} \ A_{.27a}) \ V0x) \ (c_{.2Epred\_set\_2EEMPTY} \ A_{.27a})))))) \quad (51)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. (\forall V1y \in \\ A_{.27a}. (\forall V2s \in 2^{A_{.27a}}. ((p \ (ap \ (ap \ (c_{.2Ebool\_2EIN} \ A_{.27a}) \\ V0x) \ (ap \ (ap \ (c_{.2Epred\_set\_2EINSERT} \ A_{.27a}) \ V1y) \ V2s))) \Leftrightarrow ((V0x = \\ V1y) \vee (p \ (ap \ (ap \ (c_{.2Ebool\_2EIN} \ A_{.27a}) \ V0x) \ V2s)))))) \quad (52)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0P \in (2^{(2^{A_{.27a}})}). (( \\ (p \ (ap \ V0P \ (c_{.2Epred\_set\_2EEMPTY} \ A_{.27a}))) \wedge (\forall V1s \in (2^{A_{.27a}}). \\ (((p \ (ap \ (c_{.2Epred\_set\_2EFINITE} \ A_{.27a}) \ V1s)) \wedge (p \ (ap \ V0P \ V1s))) \Rightarrow \\ (\forall V2e \in A_{.27a}. ((\neg(p \ (ap \ (ap \ (c_{.2Ebool\_2EIN} \ A_{.27a}) \ V2e) \ V1s))) \Rightarrow \\ (p \ (ap \ V0P \ (ap \ (ap \ (c_{.2Epred\_set\_2EINSERT} \ A_{.27a}) \ V2e) \ V1s)))))) \Rightarrow \\ (\forall V3s \in (2^{A_{.27a}}). ((p \ (ap \ (c_{.2Epred\_set\_2EFINITE} \ A_{.27a}) \\ V3s)) \Rightarrow (p \ (ap \ V0P \ V3s)))))) \quad (53)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{A-27a}).((ap\ (ap\ (c\_2Eproduct\_2Eproduct \\
& A.27a)\ (c\_2Epred\_set\_2EEMPTY\ A.27a))\ V0f) = (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))))) \wedge \\
& (\forall V1x \in A.27b.(\forall V2f \in (ty\_2Erealax\_2Ereal^{A-27b}). \\
& (\forall V3s \in (2^{A-27b}).((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A.27b) \\
& V3s)) \Rightarrow ((ap\ (ap\ (c\_2Eproduct\_2Eproduct\ A.27b)\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT \\
& A.27b)\ V1x)\ V3s))\ V2f) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ ty\_2Erealax\_2Ereal) \\
& (ap\ (ap\ (c\_2Ebool\_2EIN\ A.27b)\ V1x)\ V3s))\ (ap\ (ap\ (c\_2Eproduct\_2Eproduct \\
& A.27b)\ V3s)\ V2f))\ (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ (ap\ V2f\ V1x))\ ( \\
& ap\ (ap\ (c\_2Eproduct\_2Eproduct\ A.27b)\ V3s)\ V2f)))))))))) \\
& \hspace{15em} (54)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty\_2Erealax\_2Ereal^{A-27a}). \\
& (\forall V1s \in (2^{A-27a}).(((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A.27a) \\
& V1s)) \wedge (\forall V2x \in A.27a.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A.27a)\ V2x) \\
& V1s)) \Rightarrow (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))\ (ap\ V0f\ V2x)))))) \Rightarrow (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ (ap\ (ap\ (c\_2Eproduct\_2Eproduct \\
& A.27a)\ V1s)\ V0f)))))) \\
& \hspace{15em} (55)
\end{aligned}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ V0x)\ V0x))) \quad (56)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in ty\_2Erealax\_2Ereal.(\forall V1x2 \in ty\_2Erealax\_2Ereal. \\
& (\forall V2y1 \in ty\_2Erealax\_2Ereal.(\forall V3y2 \in ty\_2Erealax\_2Ereal. \\
& (((p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))\ V0x1)) \wedge ((p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))\ V2y1)) \wedge ((p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ V0x1)\ V1x2)) \wedge \\
& (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ V2y1)\ V3y2)))))) \Rightarrow (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\
& (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ V0x1)\ V2y1))\ (ap\ (ap\ c\_2Erealax\_2Ereal\_mul \\
& V1x2)\ V3y2)))))) \\
& \hspace{15em} (57)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (58)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (59)$$



Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (60)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg((p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (61)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (69)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0g \in (ty\_2Erealax\_2Ereal^{A\_27a}). \\ & \quad (\forall V1f \in (ty\_2Erealax\_2Ereal^{A\_27a}). (\forall V2s \in (2^{A\_27a}). \\ & \quad ((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ V2s)) \wedge (\forall V3x \in A\_27a. \\ (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V2s)) \Rightarrow ((p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ (ap\ V1f\ V3x))) \wedge (p \\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ (ap\ V1f\ V3x))\ (ap\ V0g\ V3x)))))) \Rightarrow ( \\ p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ (ap\ (ap\ (c\_2Eproduct\_2Eproduct \\ A\_27a)\ V2s)\ V1f))\ (ap\ (ap\ (c\_2Eproduct\_2Eproduct\ A\_27a)\ V2s)\ V0g)))))) \end{aligned}$$