

thm_2Eproduct_2EPRODUCT__MUL__NUMSEG (TMc6UUNnZz81zc1RiUPSve83Tonpkf7XmSN)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P))$

Definition 4 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_EF))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num (c_2Enum_2ESUC_REP m))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_Emin_2E_40$

Definition 11 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 13 We define $c_Earithmic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (5)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (6)$$

Definition 14 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (7)$$

Definition 15 We define $c_2Eiterate_2E_2E_2E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (8)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (9)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (10)$$

Definition 16 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap\ (c_Emin_2E_40\ (t$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (11)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (12)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (13)$$

Definition 17 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 18 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 19 We define $c_2Eiterate_2Eneutral$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap\ (c_2Emin\ A_27a\ op))$

Definition 20 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 21 We define $c_2Eiterate_2ESupport$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V1$

Definition 22 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(\lambda V3$

Definition 23 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2Emin\ A_27a\ s))$

Definition 24 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E2EF)$.

Definition 25 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E2E21\ A_27a\ s))$

Definition 26 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27b}).\lambda V1$

Definition 27 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V1$

Definition 28 We define $c_2Eproduct_2Eproduct$ to be $\lambda A_27a : \iota.(ap\ (c_2Eiterate_2Eiterate\ A_27a\ ty_2Erealax_2Ereal_ABS_CLASS))$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \quad (17) \end{aligned}$$

Assume the following.

$$p \left(\text{ap} \left(\text{c_2Epred_set_2EFINITE } ty_2Enum_2Enum \right) \left(\text{ap} \left(\text{ap } c_2Eiterate_2E_2E_2E \right. \right. \right. \\ \left. \left. \left. V0m \right) V1n \right) \right) \right) \tag{18}$$

Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0f \in (ty_2Erealax_2Ereal^{A_27a}). \\ (\forall V1g \in (ty_2Erealax_2Ereal^{A_27a}). (\forall V2s \in (2^{A_27a}). \\ ((p \left(\text{ap} \left(\text{c_2Epred_set_2EFINITE } A_27a \right) V2s \right) \Rightarrow ((\text{ap} \left(\text{ap} \left(\text{c_2Eproduct_2Eproduct} \right. \right. \right. \\ \left. \left. \left. A_27a \right) V2s \right) (\lambda V3x \in A_27a. (\text{ap} \left(\text{ap } c_2Erealax_2Ereal_mul \left(\text{ap} \right. \right. \right. \\ \left. \left. \left. V0f \ V3x \right) \right) \left(\text{ap } V1g \ V3x \right) \right) \right) = (\text{ap} \left(\text{ap } c_2Erealax_2Ereal_mul \left(\text{ap} \left(\text{ap} \right. \right. \right. \\ \left. \left. \left. \text{c_2Eproduct_2Eproduct } A_27a \right) V2s \right) V0f \right) \left(\text{ap} \left(\text{ap} \left(\text{c_2Eproduct_2Eproduct} \right. \right. \right. \\ \left. \left. \left. A_27a \right) V2s \right) V1g \right) \right) \right) \right) \tag{19}$$

Theorem 1

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). (\forall V1g \in \\ (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). (\forall V2m \in ty_2Enum_2Enum. \\ (\forall V3n \in ty_2Enum_2Enum. ((\text{ap} \left(\text{ap} \left(\text{c_2Eproduct_2Eproduct} \right. \right. \right. \\ ty_2Enum_2Enum \right) \left(\text{ap} \left(\text{ap } c_2Eiterate_2E_2E_2E \right) V2m \right) V3n \right) (\lambda V4x \in \\ ty_2Enum_2Enum. (\text{ap} \left(\text{ap } c_2Erealax_2Ereal_mul \left(\text{ap } V0f \ V4x \right) \right) \left(\right. \\ \left. \text{ap } V1g \ V4x \right) \right) \right) = (\text{ap} \left(\text{ap } c_2Erealax_2Ereal_mul \left(\text{ap} \left(\text{ap} \left(\text{c_2Eproduct_2Eproduct} \right. \right. \right. \\ ty_2Enum_2Enum \right) \left(\text{ap} \left(\text{ap } c_2Eiterate_2E_2E_2E \right) V2m \right) V3n \right) V0f \right) \\ \left(\text{ap} \left(\text{ap} \left(\text{c_2Eproduct_2Eproduct } ty_2Enum_2Enum \right) \left(\text{ap} \left(\text{ap } c_2Eiterate_2E_2E_2E \right. \right. \right. \\ \left. \left. \left. V2m \right) V3n \right) \right) V1g \right) \right) \right) \right)$$