

# thm\_2Eproduct\_2EPRODUCT\_POS\_LE (Tmb- JUYwBCaxxguEyUYgGk52mbFx4XQnfRRx)

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**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.$ **if**  $(\exists x \in A.p (ap P x))$  **then** *(the*  $(\lambda x.x \in A \wedge p)$  *of type*  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  *of type*  $\iota \Rightarrow \iota$ .

**Definition 3** We define `c_2Ebool_2E_3F` to be  $\lambda A_{.27a} : \iota.(\lambda V0P \in (2^{A_{.27a}}).(ap V0P (ap (c_2Emin_2E_40 A_{.27a} P))))$

**Definition 4** We define `c_2Ebool_2E_2T` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 5** We define `c_2Ebool_2E_2IN` to be  $\lambda A_{.27a} : \iota.(\lambda V0x \in A_{.27a}.(\lambda V1f \in (2^{A_{.27a}}).(ap V1f V0x)))$

**Definition 6** We define `c_2Ebool_2E_21` to be  $\lambda A_{.27a} : \iota.(\lambda V0P \in (2^{A_{.27a}}).(ap (ap (c_2Emin_2E_3D (2^{A_{.27a}}) P) P)))$

**Definition 7** We define `c_2Ebool_2E_2F` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 8** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  *of type*  $\iota$ .

**Definition 9** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

**Definition 10** We define `c_2Ebool_2E_2COND` to be  $\lambda A_{.27a} : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_{.27a}.(\lambda V2t2 \in A_{.27a}.(ap (c_2Emin_2E_3D (2^{A_{.27a}}) V2t2) V1t1))))$

**Definition 11** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let `c_2Epair_2EABS_prod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow c\_2Epair\_2EABS\_prod A_{.27a} A_{.27b} \in ((ty\_2Epair\_2Eprod A_{.27a} A_{.27b})^{(2^{A_{.27b}})^{A_{.27a}}}) \tag{2}$$

**Definition 12** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}})$$
(3)

**Definition 13** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A\_27a}). (ap (c\_2Ebool\_2E21$

**Definition 14** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2E2F)$ .

**Definition 15** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). (ap (c\_2Ebool\_2E21$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega$$
(4)

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum$$
(5)

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega})$$
(6)

**Definition 16** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 17** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum})$$
(7)

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega})$$
(8)

**Definition 18** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(9)

**Definition 19** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap (ap c\_2Earithmetic$

**Definition 20** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (10)$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (11)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (12)$$

**Definition 21** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E40\ t$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (13)$$

Let  $c\_2Erealax\_2Etrealeq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealeq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (14)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (15)$$

**Definition 22** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 23** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 24** We define  $c\_2Eiterate\_2Eneutral$  to be  $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (c\_2Emin$

**Definition 25** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2).(ap\ (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E7E$

**Definition 26** We define  $c\_2Eiterate\_2Esupport$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

**Definition 27** We define  $c\_2Eiterate\_2EITSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}).\lambda V$

**Definition 28** We define  $c\_2Eiterate\_2Eiterate$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

**Definition 29** We define  $c\_2Eproduct\_2Eproduct$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Eiterate\_2Eiterate\ A\_27a\ ty\_2Ereal$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal)^{ty\_2Eenum\_2Eenum} \quad (16)$$

Let  $c\_2Erealax\_2Etreallt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreallt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (17)$$

**Definition 30** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 31** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (23)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (24)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF) \\ & V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in \\ & 2. (((\forall V2x \in A\_27a. (p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in \\ & A\_27a. ((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee ( \\ & (p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\ & V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\ & V5y\_27)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0r \in 2. (\forall V1p \in 2. (\forall V2q \in 2. (((p\ V1p) \wedge \\ & (p\ V2q) \Rightarrow (p\ V0r)) \Leftrightarrow ((p\ V1p) \Rightarrow ((p\ V2q) \Rightarrow (p\ V0r)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\ & A\_27a. (\forall V2s \in (2^{A\_27a}). ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\ & V0x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ A\_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = \\ & V1y) \vee (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ V2s)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(2^{A-27a})}).(( \\
& \quad (p\ (ap\ V0P\ (c.2Epred\_set\_2EEMPTY\ A.27a))) \wedge (\forall V1s \in (2^{A-27a}). \\
& \quad ((p\ (ap\ (c.2Epred\_set\_2EFINITE\ A.27a)\ V1s)) \wedge (p\ (ap\ V0P\ V1s))) \Rightarrow \\
& \quad (\forall V2e \in A.27a.((\neg(p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a)\ V2e)\ V1s))) \Rightarrow \\
& \quad (p\ (ap\ V0P\ (ap\ (ap\ (c.2Epred\_set\_2EINSERT\ A.27a)\ V2e)\ V1s)))))) \Rightarrow \\
& \quad (\forall V3s \in (2^{A-27a}).((p\ (ap\ (c.2Epred\_set\_2EFINITE\ A.27a) \\
& \quad V3s)) \Rightarrow (p\ (ap\ V0P\ V3s))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\
& \quad (\forall V0f \in (ty\_2Erealax\_2Ereal^{A-27a}).((ap\ (ap\ (c.2Eproduct\_2Eproduct \\
& \quad A.27a)\ (c.2Epred\_set\_2EEMPTY\ A.27a))\ V0f) = (ap\ c.2Ereal\_2Ereal\_of\_num \\
& \quad (ap\ c.2Earithmetic\_2ENUMERAL\ (ap\ c.2Earithmetic\_2EBIT1\ c.2Earithmetic\_2EZERO)))))) \wedge \\
& \quad (\forall V1x \in A.27b.(\forall V2f \in (ty\_2Erealax\_2Ereal^{A-27b}). \\
& \quad (\forall V3s \in (2^{A-27b}).((p\ (ap\ (c.2Epred\_set\_2EFINITE\ A.27b) \\
& \quad V3s)) \Rightarrow ((ap\ (ap\ (c.2Eproduct\_2Eproduct\ A.27b)\ (ap\ (ap\ (c.2Epred\_set\_2EINSERT \\
& \quad A.27b)\ V1x)\ V3s))\ V2f) = (ap\ (ap\ (ap\ (c.2Ebool\_2ECOND\ ty\_2Erealax\_2Ereal) \\
& \quad (ap\ (ap\ (c.2Ebool\_2EIN\ A.27b)\ V1x)\ V3s))\ (ap\ (ap\ (c.2Eproduct\_2Eproduct \\
& \quad A.27b)\ V3s)\ V2f))\ (ap\ (ap\ c.2Erealax\_2Ereal\_mul\ (ap\ V2f\ V1x))\ ( \\
& \quad ap\ (ap\ (c.2Eproduct\_2Eproduct\ A.27b)\ V3s)\ V2f))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& \quad (((p\ (ap\ (ap\ c.2Ereal\_2Ereal\_lte\ (ap\ c.2Ereal\_2Ereal\_of\_num \\
& \quad c.2Enum\_2E0))\ V0x)) \wedge (p\ (ap\ (ap\ c.2Ereal\_2Ereal\_lte\ (ap\ c.2Ereal\_2Ereal\_of\_num \\
& \quad c.2Enum\_2E0))\ V1y))) \Rightarrow (p\ (ap\ (ap\ c.2Ereal\_2Ereal\_lte\ (ap\ c.2Ereal\_2Ereal\_of\_num \\
& \quad c.2Enum\_2E0))\ (ap\ (ap\ c.2Erealax\_2Ereal\_mul\ V0x)\ V1y))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c.2Ereal\_2Ereal\_lte \\
& \quad (ap\ c.2Ereal\_2Ereal\_of\_num\ c.2Enum\_2E0))\ (ap\ c.2Ereal\_2Ereal\_of\_num \\
& \quad V0n))))
\end{aligned} \tag{39}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{40}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{42}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg( \\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ( \\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (51)$$

**Theorem 1**

$$\begin{aligned} \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0f \in (\text{ty\_2Ereal} \times \text{2Ereal}^{A_{27a}}). \\ (\forall V1s \in (2^{A_{27a}}). ((p \text{ (ap (c\_2Epred\_set\_2EFINITE } A_{27a}) \\ V1s)) \wedge (\forall V2x \in A_{27a}. ((p \text{ (ap (ap (c\_2Ebool\_2EIN } A_{27a}) V2x) \\ V1s)) \Rightarrow (p \text{ (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num} \\ \text{c\_2Enum\_2E0)) (ap V0f V2x)))))) \Rightarrow (p \text{ (ap (ap c\_2Ereal\_2Ereal\_lte} \\ \text{(ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) (ap (ap (c\_2Eproduct\_2Eproduct} \\ \text{A\_27a) V1s) V0f))))))) \end{aligned}$$