

# thm\_2EquantHeuristics\_2EFST\_\_PAIR\_\_EQ (TMWDTf4mRcv3LnA3pz6mxYZ6jFQkEpdJKe3)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2ET` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define `c_2Ebool_2E_21` to be  $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

**Definition 4** We define `c_2Ebool_2EF` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

**Definition 7** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define `c_2Ebool_2E_3F` to be  $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a) (\lambda V1P \in 2.V1P))))$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let `c_2Epair_2EFST` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c\_2Epair\_2EFST A_27a A_27b \in (A_27a (ty\_2Epair\_2Eprod A_27a A_27b)) \tag{2}$$

Let `c_2Epair_2EABS_prod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c\_2Epair\_2EABS\_prod A_27a A_27b \in ((ty\_2Epair\_2Eprod A_27a A_27b)^{(2^{A-27b})^{A-27a}}) \tag{3}$$

**Definition 9** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c\_2E$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c\_2Epair\_2ESND \\ A.27a A.27b \in (A.27b^{(ty\_2Epair\_2Eprod A.27a A.27b)}) \end{aligned} \quad (4)$$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A.27a.(p V0t)) \Leftrightarrow (p V0t))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow \\ True)) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\ \forall V0x \in A.27a.(\forall V1y \in A.27b.(\forall V2a \in A.27a.(\forall V3b \in \\ A.27b.(((ap (ap (c\_2Epair\_2E\_2C A.27a A.27b) V0x) V1y) = (ap (ap \\ (c\_2Epair\_2E\_2C A.27a A.27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\ \forall V0x \in (ty\_2Epair\_2Eprod A.27a A.27b).(\exists V1q \in A.27a. \\ (\exists V2r \in A.27b.(V0x = (ap (ap (c\_2Epair\_2E\_2C A.27a A.27b) \\ V1q) V2r)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\ \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap (c\_2Epair\_2EFST A.27a \\ A.27b) (ap (ap (c\_2Epair\_2E\_2C A.27a A.27b) V0x) V1y)) = V0x))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\ \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap (c\_2Epair\_2ESND A.27a \\ A.27b) (ap (ap (c\_2Epair\_2E\_2C A.27a A.27b) V0x) V1y)) = V1y))) \end{aligned} \quad (12)$$

**Theorem 1**

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow \forall A_{27b}. \text{nonempty } A_{27b} \Rightarrow ( \\ & \quad \forall V_0p \in (\text{ty\_2Epair\_2Eprod } A_{27a} \ A_{27b}). (\forall V_1p_2 \in A_{27b}. \\ & \quad ((\text{ap } (\text{ap } (\text{c\_2Epair\_2E\_2C } A_{27a} \ A_{27b}) \ (\text{ap } (\text{c\_2Epair\_2EFST } A_{27a} \\ & \quad A_{27b}) \ V_0p)) \ V_1p_2) = V_0p) \Leftrightarrow (V_1p_2 = (\text{ap } (\text{c\_2Epair\_2ESND } A_{27a} \ A_{27b}) \\ & \quad V_0p)))))) \end{aligned}$$