

thm_2EquantHeuristics_2EGUESSES_NEG_DUALITY (TMNU9WRdPoL4yiq2Zc3A1V8woaix9ReGWNy)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A-27c}).\lambda V1g$

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 7 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 10 We define $c_2EquantHeuristics_2EGUESS_FORALL_GAP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A-27a}).\lambda V1P \in (2^{A-27b}).(ap (c_2Ebool_2E_21 A_27b) (\lambda V2v \in A_27b.(a$

Definition 11 We define $c_2EquantHeuristics_2EGUESS_EXISTS_GAP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A-27a}).\lambda V1P \in (2^{A-27b}).(ap (c_2Ebool_2E_21 A_27b) (\lambda V2v \in A_27b.(a$

Definition 12 We define $c_2EquantHeuristics_2EGUESS_FORALL_POINT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A-27a}).\lambda V1P \in (2^{A-27b}).(ap (c_2Ebool_2E_21 A_27a) (\lambda V2fv \in A_27a.(a$

Definition 13 We define $c_2EquantHeuristics_2EGUESS_EXISTS_POINT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A-27a}).\lambda V1P \in (2^{A-27b}).(ap (c_2Ebool_2E_21 A_27a) (\lambda V2fv \in A_27a.(a$

Definition 14 We define $c_2EquantHeuristics_2EGUESS_FORALL$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A-27a}).\lambda V1P \in (2^{A-27b}).(ap (c_2Ebool_2E_7E A_27a) (\lambda V2fv \in A_27a.(a$

Definition 15 We define `c_2EquantHeuristics_2EGUESS_EXISTS` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda V0i \in (A_{.27a} \dots$

Definition 16 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \tag{3}$$

Assume the following.

$$\forall A_{.27a}. nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. ((V0x = V0x) \Leftrightarrow True)) \tag{4}$$

Assume the following.

$$\forall A_{.27a}. nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. (\forall V1y \in A_{.27a}. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{5}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in \\ & 2. (((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. nonempty A_{.27a} \Rightarrow \forall A_{.27b}. nonempty A_{.27b} \Rightarrow \forall A_{.27c}. \\ & nonempty A_{.27c} \Rightarrow (\forall V0f \in (A_{.27b}^{A_{.27a}}). (\forall V1g \in (A_{.27a}^{A_{.27c}}). \\ & (\forall V2x \in A_{.27c}. ((ap (ap (ap (c_2Ecombin_2Eo A_{.27c} A_{.27b} A_{.27a}) \\ & V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \end{aligned} \tag{8}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0i \in (A_27b^{A_27a}). (\forall V1P \in (2^{A_27b}). (((p\ (ap\ (\\
& \quad ap\ (c_2EquantHeuristics_2EGUESS_EXISTS\ A_27a\ A_27b)\ V0i)\ V1P)) \Leftrightarrow \\
& \quad (\forall V2v \in A_27b. ((p\ (ap\ V1P\ V2v)) \Rightarrow (\exists V3fv \in A_27a. (p\ (\\
& \quad ap\ V1P\ (ap\ V0i\ V3fv)))))) \wedge ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL \\
& \quad A_27a\ A_27b)\ V0i)\ V1P)) \Leftrightarrow (\forall V4v \in A_27b. ((\neg(p\ (ap\ V1P\ V4v))) \Rightarrow \\
& \quad (\exists V5fv \in A_27a. (\neg(p\ (ap\ V1P\ (ap\ V0i\ V5fv)))))) \wedge ((\forall V6i \in \\
& \quad (A_27b^{A_27a}). (\forall V7P \in (2^{A_27b}). ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_POINT \\
& \quad A_27a\ A_27b)\ V6i)\ V7P)) \Leftrightarrow (\forall V8fv \in A_27a. (p\ (ap\ V7P\ (ap\ V6i\ V8fv)))))) \wedge \\
& \quad ((\forall V9i \in (A_27b^{A_27a}). (\forall V10P \in (2^{A_27b}). ((p\ (ap \\
& \quad (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_POINT\ A_27a\ A_27b) \\
& \quad V9i)\ V10P)) \Leftrightarrow (\forall V11fv \in A_27a. (\neg(p\ (ap\ V10P\ (ap\ V9i\ V11fv)))))) \wedge \\
& \quad ((\forall V12i \in (A_27b^{A_27a}). (\forall V13P \in (2^{A_27b}). ((p\ (ap \\
& \quad (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_GAP\ A_27a\ A_27b) \\
& \quad V12i)\ V13P)) \Leftrightarrow (\forall V14v \in A_27b. ((p\ (ap\ V13P\ V14v)) \Rightarrow (\exists V15fv \in \\
& \quad A_27a. (V14v = (ap\ V12i\ V15fv)))))) \wedge (\forall V16i \in (A_27b^{A_27a}). \\
& \quad (\forall V17P \in (2^{A_27b}). ((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_GAP \\
& \quad A_27a\ A_27b)\ V16i)\ V17P)) \Leftrightarrow (\forall V18v \in A_27b. ((\neg(p\ (ap\ V17P\ V18v))) \Rightarrow \\
& \quad (\exists V19fv \in A_27a. (V18v = (ap\ V16i\ V19fv))))))))) \\
& \hspace{15em} (9)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0i \in (A_27b^{A_27a}). (\forall V1P \in (2^{A_27b}). (((p\ (ap\ (ap \\
& \quad (c_2EquantHeuristics_2EGUESS_EXISTS\ A_27a\ A_27b)\ V0i)\ (ap\ (\\
& \quad ap\ (c_2Ecombin_2Eo\ A_27b\ 2\ 2)\ c_2Ebool_2E_7E)\ V1P)) \Leftrightarrow (p\ (ap\ (\\
& \quad ap\ (c_2EquantHeuristics_2EGUESS_FORALL\ A_27a\ A_27b)\ V0i)\ V1P))) \wedge \\
& \quad (((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL\ A_27a\ A_27b) \\
& \quad V0i)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27b\ 2\ 2)\ c_2Ebool_2E_7E)\ V1P))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS\ A_27a\ A_27b) \\
& \quad V0i)\ V1P))) \wedge (((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_GAP \\
& \quad A_27a\ A_27b)\ V0i)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27b\ 2\ 2)\ c_2Ebool_2E_7E) \\
& \quad V1P))) \Leftrightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_GAP \\
& \quad A_27a\ A_27b)\ V0i)\ V1P))) \wedge (((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_GAP \\
& \quad A_27a\ A_27b)\ V0i)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27b\ 2\ 2)\ c_2Ebool_2E_7E) \\
& \quad V1P))) \Leftrightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_GAP \\
& \quad A_27a\ A_27b)\ V0i)\ V1P))) \wedge (((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_POINT \\
& \quad A_27a\ A_27b)\ V0i)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27b\ 2\ 2)\ c_2Ebool_2E_7E) \\
& \quad V1P))) \Leftrightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_POINT \\
& \quad A_27a\ A_27b)\ V0i)\ V1P))) \wedge (((p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_FORALL_POINT \\
& \quad A_27a\ A_27b)\ V0i)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27b\ 2\ 2)\ c_2Ebool_2E_7E) \\
& \quad V1P))) \Leftrightarrow (p\ (ap\ (ap\ (c_2EquantHeuristics_2EGUESS_EXISTS_POINT \\
& \quad A_27a\ A_27b)\ V0i)\ V1P))))))))) \\
\end{aligned}$$