thm_2EquantHeuristics_2EGUESSES__UEXISTS__THM1 (TMH8f1w5tZimzsfPgnjSLQRbUvk8Bt4vcaP)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Definition 7 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40 A

Definition 8 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ c_2Ebool_2E_2F_5C

Definition 9 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 11 *We define* c_2EquantHeuristics_2EGUESS__FORALL__GAP *to be*
$\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap\ ($c_2Ebool_2E_21 $A\_27b)\ (\lambda V2v \in A\_27b.(a$

Definition 12 *We define* c_2EquantHeuristics_2EGUESS__EXISTS__GAP *to be*
$\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap\ ($c_2Ebool_2E_21 $A\_27b)\ (\lambda V2v \in A\_27b.(a$

Definition 13 *We define* c_2EquantHeuristics_2EGUESS__FORALL__POINT *to be*
$\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap\ ($c_2Ebool_2E_21 $A\_27a)\ (\lambda V2fv \in A\_2$

Definition 14 *We define* c_2EquantHeuristics_2EGUESS__EXISTS__POINT *to be*
$\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap\ ($c_2Ebool_2E_21 $A\_27a)\ (\lambda V2fv \in A\_27a.$

Definition 15 *We define* c_2EquantHeuristics_2EGUESS__FORALL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in (A\_27$

Definition 16 *We define* c_2EquantHeuristics_2EGUESS__EXISTS *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in (A\_27$

Definition 17 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{2}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{3}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{4}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \tag{5}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{6}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{10}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P)\wedge(\forall V2x \in A\_27a.(p\ (ap\ V1Q\ V2x))))\Leftrightarrow(\forall V3x \in A\_27a.((p\ V0P)\wedge(p\ (ap\ V1Q\ V3x))))))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0Q \in 2.(\forall V1P \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V1P\ V2x))\vee(p\ V0Q)))\Leftrightarrow((\forall V3x \in A\_27a.(p\ (ap\ V1P\ V3x)))\vee(p\ V0Q))))) \tag{13}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.((\neg((p\ V0A)\Rightarrow(p\ V1B)))\Leftrightarrow((p\ V0A)\wedge (\neg(p\ V1B)))))) \tag{14}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A)\vee((p\ V1B)\wedge(p\ V2C)))\Leftrightarrow(((p\ V0A)\vee(p\ V1B))\wedge((p\ V0A)\vee(p\ V2C))))))) \tag{15}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F\_21}\ A\_27a)\ (\lambda V1x \in A\_27a.(ap\ V0P\ V1x))))\Leftrightarrow((\exists V2x \in A\_27a.(p\ (ap\ V0P\ V2x)))\wedge(\forall V3x \in A\_27a.(\forall V4y \in A\_27a.(((p\ (ap\ V0P\ V3x))\wedge(p\ (ap\ V0P\ V4y)))\Rightarrow(V3x = V4y))))))) \tag{17}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0i \in (A\_27b^{A\_27a}).(\forall V1P \in (2^{A\_27b}).((((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS}\ A\_27a\ A\_27b)\ V0i)\ V1P)) \Leftrightarrow (\forall V2v \in A\_27b.((p\ (ap\ V1P\ V2v)) \Rightarrow (\exists V3fv \in A\_27a.(p\ (ap\ V1P\ (ap\ V0i\ V3fv))))))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL}\ A\_27a\ A\_27b)\ V0i)\ V1P)) \Leftrightarrow (\forall V4v \in A\_27b.((\neg(p\ (ap\ V1P\ V4v))) \Rightarrow (\exists V5fv \in A\_27a.(\neg(p\ (ap\ V1P\ (ap\ V0i\ V5fv))))))))) \wedge ((\forall V6i \in (A\_27b^{A\_27a}).(\forall V7P \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS\_\_POINT}\ A\_27a\ A\_27b)\ V6i)\ V7P)) \Leftrightarrow (\forall V8fv \in A\_27a.(p\ (ap\ V7P\ (ap\ V6i\ V8fv))))))) \wedge ((\forall V9i \in (A\_27b^{A\_27a}).(\forall V10P \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL\_\_POINT}\ A\_27a\ A\_27b)\ V9i)\ V10P)) \Leftrightarrow (\forall V11fv \in A\_27a.(\neg(p\ (ap\ V10P\ (ap\ V9i\ V11fv)))))))) \wedge ((\forall V12i \in (A\_27b^{A\_27a}).(\forall V13P \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS\_\_GAP}\ A\_27a\ A\_27b)\ V12i)\ V13P)) \Leftrightarrow (\forall V14v \in A\_27b.((p\ (ap\ V13P\ V14v)) \Rightarrow (\exists V15fv \in A\_27a.(V14v = (ap\ V12i\ V15fv)))))))) \wedge (\forall V16i \in (A\_27b^{A\_27a}). (\forall V17P \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL\_\_GAP}\ A\_27a\ A\_27b)\ V16i)\ V17P)) \Leftrightarrow (\forall V18v \in A\_27b.((\neg(p\ (ap\ V17P\ V18v))) \Rightarrow (\exists V19fv \in A\_27a.(V18v = (ap\ V16i\ V19fv))))))))))))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{20}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{21}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{22}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{23}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{24}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \tag{25}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{26}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{27}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{28}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{29}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{30}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{31}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{32}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{33}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{34}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0i \in A\_27a.(\forall V1P \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS}\ A\_27b\ A\_27a)\ (\lambda V2x \in A\_27b.V0i))\ V1P))\Rightarrow((p\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F\_21}\ A\_27a)\ V1P))\Leftrightarrow((p\ (ap\ V1P\ V0i))\wedge(\forall V3v \in A\_27a.((p\ (ap\ V1P\ V3v))\Rightarrow(V3v = V0i))))))))$$