thm_2EquantHeuristics_2EGUESSES__WEAKEN__THM (TMW8QKQoaXiBRbuuRZkpfwRbxfGFRbKwwCX)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^{2}))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40 A

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Definition 9 *We define* c_2EquantHeuristics_2EGUESS__FORALL__GAP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in$

Definition 10 *We define* c_2EquantHeuristics_2EGUESS__EXISTS__GAP *to be*
$\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap\ ($c_2Ebool_2E_21 $A\_27b)\ (\lambda V2v \in A\_27b.(a$

Definition 11 *We define* c_2EquantHeuristics_2EGUESS__FORALL__POINT *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap\ ($c_2Ebool_2E_21 $A\_27a)\ (\lambda V2fv \in A\_27$

Definition 12 *We define* c_2EquantHeuristics_2EGUESS__EXISTS__POINT *to be*
$\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in (A_27b^{A_27a}).\lambda V1P \in (2^{A_27b}).(ap\ ($c_2Ebool_2E_21 $A\_27a)\ (\lambda V2fv \in A\_27a.$

Definition 13 *We define* c_2EquantHeuristics_2EGUESS__FORALL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in (A\_2$

Definition 14 *We define* c_2EquantHeuristics_2EGUESS__EXISTS *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in (A\_27$

Definition 15 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 16 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{2}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{3}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{5}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{7}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{9}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{10}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{11}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0i \in (A\_27b^{A\_27a}).(\forall V1P \in (2^{A\_27b}).((((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS}\ A\_27a\ A\_27b)\ V0i)\ V1P))\Leftrightarrow (\forall V2v \in A\_27b.((p\ (ap\ V1P\ V2v))\Rightarrow(\exists V3fv \in A\_27a.(p\ (ap\ V1P\ (ap\ V0i\ V3fv)))))))\wedge((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL}\ A\_27a\ A\_27b)\ V0i)\ V1P))\Leftrightarrow(\forall V4v \in A\_27b.((\neg(p\ (ap\ V1P\ V4v)))\Rightarrow (\exists V5fv \in A\_27a.(\neg(p\ (ap\ V1P\ (ap\ V0i\ V5fv)))))))))\wedge((\forall V6i \in (A\_27b^{A\_27a}).(\forall V7P \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS\_\_POINT}\ A\_27a\ A\_27b)\ V6i)\ V7P))\Leftrightarrow(\forall V8fv \in A\_27a.(p\ (ap\ V7P\ (ap\ V6i\ V8fv))))))))\wedge ((\forall V9i \in (A\_27b^{A\_27a}).(\forall V10P \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL\_\_POINT}\ A\_27a\ A\_27b)\ V9i)\ V10P))\Leftrightarrow(\forall V11fv \in A\_27a.(\neg(p\ (ap\ V10P\ (ap\ V9i\ V11fv))))))))\wedge ((\forall V12i \in (A\_27b^{A\_27a}).(\forall V13P \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS\_\_GAP}\ A\_27a\ A\_27b)\ V12i)\ V13P))\Leftrightarrow(\forall V14v \in A\_27b.((p\ (ap\ V13P\ V14v))\Rightarrow(\exists V15fv \in A\_27a.(V14v = (ap\ V12i\ V15fv))))))))\wedge(\forall V16i \in (A\_27b^{A\_27a}). (\forall V17P \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL\_\_GAP}\ A\_27a\ A\_27b)\ V16i)\ V17P))\Leftrightarrow(\forall V18v \in A\_27b.((\neg(p\ (ap\ V17P\ V18v)))\Rightarrow (\exists V19fv \in A\_27a.(V18v = (ap\ V16i\ V19fv))))))))))))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{14}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{15}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow (((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{16}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{17}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{18}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \tag{19}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \tag{20}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \tag{21}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))) \tag{22}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \tag{23}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \tag{24}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg(p\ V0p))))) \tag{25}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \tag{26}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p))) \Rightarrow (p\ V0p))) \tag{27}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0i \in (A\_27b^{A\_27a}).(\forall V1P \in (2^{A\_27b}).(((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL\_\_GAP}\ A\_27a\ A\_27b)\ V0i)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL}\ A\_27a\ A\_27b)\ V0i)\ V1P))) \wedge (((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL\_\_POINT}\ A\_27a\ A\_27b)\ V0i)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_FORALL}\ A\_27a\ A\_27b)\ V0i)\ V1P))) \wedge (((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS\_\_POINT}\ A\_27a\ A\_27b)\ V0i)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS}\ A\_27a\ A\_27b)\ V0i)\ V1P))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS\_\_GAP}\ A\_27a\ A\_27b)\ V0i)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2EquantHeuristics\_2EGUESS\_\_EXISTS}\ A\_27a\ A\_27b)\ V0i)\ V1P))))))))$$